International Journal of Applied Information Systems (IJAIS) ISSN : 2249-0868
Foundation of Computer Science FCS, New York, USA
Volume 7 - No. 7, August 2014 - www.ijais.org

Intuitionistic Fuzzy Reliability of k-out-of-n System using Statistical Confidence Interval

Gaurav Kumar
Research Scholar,
Singhania University,
Pacheri Bari, Rajasthan, India

Rakesh K Bajaj,
Associate Professor, Department of Mathematics,
JUIT, Waknaghat,
Solan, Himachal Pradesh, India

ABSTRACT

In the present communication, some new arithmetic operations on intuitionistic fuzzy numbers using the α-cut method are introduced. A new methodology based on intuitionistic fuzzy confidence interval has been provided for analyzing the intuitionistic fuzzy system reliability of k-out-of-n system (particularly, series and parallel system), where the reliability of each component of each system is unknown. The reliability of each component of the system using the intuitionistic fuzzy statistical sample data using the α-cuts of $(1-\gamma)100\%$ approach is estimated to compute the system reliability. Further, based on the estimated reliability of the components obtained, the intuitionistic fuzzy reliability of the system has been finally calculated using the minimal path sets approach.

Keywords:

Reliability Engineering; Fuzzy Reliability; Trapezoidal Intuitionistic Fuzzy Number; k-out-of-n System; Series and Parallel Systems.

1. INTRODUCTION

The reliability analysis is an important research topic in the field of engineering and science. Several researchers pay attention to elaborate the concept of reliability analysis. In reliability theory, a k-out-of-n system consists of n components of the same kind with independent and isotropic distributed lifetimes. And this kind of statistics has found wide range of applications in many industrial processes and other applied areas. A n components system works if and only if at least k of the n- components works is called a k-out of n:G system and n-component system that fails if and only if at least k of the n- components fail is called a k-out of n:F system. Therefore k-out of n:G system is equivalent to an $n-k+1$-out of n:F system. The k-out of n system structure is a very popular type of redundancy in fault tolerant systems.
The reliability of a system can be determined on the basis of tests or the acquisition of operational data. However, due to uncertainty and inaccuracy of this data, the estimation of precise values of probabilities is very difficult in many system. To overcome this problem, the concept of fuzzy set theory in the evaluation of the reliability of a system was proposed by Onisawa and Kacprzyk [1]. Cai *et al.* [2, 3] studied some special fuzzy system suchas parellel and series systems involving fuzzysets. Further, fuzzy set approach for fault tree and reliability analysis in which the relative frequencies of the basic events are considered as fuzzy numbers was proposed by Singer [4]. In order to analyze fuzzy system reliability Cheng and Mon [5] used interval of confidence . Chen [6] presented a new method for fuzzy system reliability analysis using fuzzy number arithmetic operations in which the reliability of each component is considered as fuzzy number and used simplified fuzzy arithmetic operations rather than complicated interval fuzzy arithmetic operations of fuzzy numbers [5] or the complicated extended algebraic fuzzy numbers [4].

After the successful applications of the fuzzy set theory since 1970, several researchers are engaged in their extensions. Out of existence of several extensions, i.e. Intuitionistic Fuzzy Sets (IFSs) [7], Interval-Valued Intuitionistic Fuzzy Sets (IVIFSs) [8], Vague Sets [9], R-Fuzzy Sets [10] and Interval-Valued Fuzzy Sets (IVFSs) [11], it may be noted that IVFS theory is equivalent to IFS theory, which in its turn is equivalent to Vague Set theory, and IVIFS theory extends IFS theory. The implementation of intuitionistic (vague) fuzzy set theory instead of fuzzy set theory means the introduction of another degree of freedom into a set description. Burillo [15] studied perturbations of intuitionistic fuzzy number and their properties of the correlation between these numbers. Further, a method to analyze the fuzzy reliability of the series and parallel system using triangular intuitionistic fuzzy numbers (TIFNs) arithmetic operations was proposed by Mahapatra and Roy [16]. Shing Yao *et al.* [17] applied a statistical methodology in fuzzy system reliability analysis.
In the present paper, section 2 presents some basics of k-out-of-n system with identical or non-identical components. Section 3, presents some basics of intuitionistic fuzzy set theory with some new proposed arithmetic operations using the α-cut method. Section 4, describes the methodology to calculated the reliability of k-out-of-n system using minimal path sets approach based on the estimated reliability of the components . In section 5, the proposed methodology has been illustrated with the help of a numerical example.

2. PRELIMINARIES

This section discusses the techniques for reliability evaluation of k-out-of-n : G with independent and identically distributed (i.i.d.) and independent and non-identically distributed (non-i.i.d.) components.

2.1 k-out-of-n System with I.I.D. Components

The system reliability R of a system with n independent and identically distributed (i.i.d.) components can be determined by component's reliability p_i, $i = 1, 2, \ldots, n$. and which is given by

$$R = \phi(p_1, p_2, \ldots, p_n). \tag{1}$$

where the structure function ϕ is decided by the structure of the system and R is a function of $p_1, p_2, \ldots, p_n$. The number of working components follows the binomial distribution with parameter (n, p) in a k-out-of-n : G system with i.i.d. components and the reliability of the system is equal to the probability that the number of working components is greater than or equal to k:

$$R_G(n, p) = \sum_{i=k}^{n} \binom{n}{i} p^i q^{n-i}. \tag{2}$$

The reliability of the series system and parallel system are given by

$$R = \prod_{i=1}^{n} p_i. \tag{3}$$

International Journal of Applied Information Systems (IJAIS) ISSN : 2249-0868
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$$R = 1 - \prod_{i=1}^{n}(1 - p_i). \tag{4}$$

2.2 k-out-of-n System with Independent and Non-I.I.D. Components

For k-out-of-n: G systems with components whose reliabilities are not necessarily identical, computing the system reliability is somewhat more difficult. However, more efficient recursive algorithms for reliability evaluation of such systems were reported by Barlow and Heidtmann [18] and Rushdi [19, 20]. The iterative implementation Rushdi algorithm is provided in [21]. The concept of minimal path sets to evaluate system reliability can also be used. The reliability of any system is equal to the probability that at least one of the minimal path sets works. The unreliability of the system is equal to the probability that at least one minimal cut set is failed. For a minimal path set to work, each component in the set must work. For a minimal cut set to fail, all components in the set must fail. In a k-out-of-n:G system, there are n_{C_k} minimal path sets and $n_{C_{n-k+1}}$ minimal cut sets. Each minimal path set contains exactly k different components and each minimal cut set contains exactly $n-k+1$ components. If all minimal path sets and minimal cut sets are known, then the reliability of k-out-of-n system is calculated using the following formula [18]:

$$Rel(k, n) = 1 - \prod_{j=1}^{n_{C_k}} \left[1 - \prod_{i \in P_j} R_i\right], \tag{5}$$

where P_j is the j^{th} minimal path set.

3. BASIC CONCEPTS OF INTUITIONISTIC FUZZY SETS

DEFINITION 1 FUZZY SET. *A fuzzy set* $A = \{\langle x, \mu(x)\rangle | x \in X\}$ *in a universe of discourse X is characterized by a membership function* μ_A *as follows [22].*

$$\mu_A : X \to [0, 1]. \tag{6}$$

DEFINITION 2 INTUITIONISTIC FUZZY SET. *[23, 7] Let X be the universe of discourse. Then an IFS* $\tilde{A}$ *in X is given by*

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle : x \in X\}, \tag{7}$$

where $\mu_{\tilde{A}} : X \to [0, 1]$ and $\nu_{\tilde{A}} : X \to [0, 1]$ with the condition $0 \le \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \le 1, \forall x \in X$. The numbers $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ denotes the degree of membership and non-membership of an element x to a set $\tilde{A}$ respectively. For each element $x \in X$, the amount $\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x)$ is called the degree of indeterminacy (hesitation part). It is the degree of uncertainty whether x belongs to $\tilde{A}$ or not.

DEFINITION 3 α-CUT OF THE IFS. *The* α*-cut of a intuitionistic fuzzy set, is a crisp set which consists of elements of* $\tilde{A}$ *for which the membership degree greater than or equal to* α *and non-membership degree less than or equal to* $1 - \alpha$ *i.e.,*

$$\tilde{A}_\alpha = \{x | \mu_{\tilde{A}}(x) \ge \alpha \, and \, \nu_{\tilde{A}}(x) \le 1 - \alpha\}, \forall \alpha \in [0, 1]. \tag{8}$$

DEFINITION 4 INTUITIONISTIC FUZZY NUMBER. *An intuitionistic fuzzy subset* $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle : x \in X\}$ *of the real line R is called an intuitionistic fuzzy number if the following axioms hold:*

(1) *$\tilde{A}$ is normal, i.e., there at least two points $x_1, x_2 \in R$ such that $\mu_{\tilde{A}}(x_1) = 1$ and $\nu_{\tilde{A}}(x_2) = 0$;*
(2) *The membership function $\mu_{\tilde{A}}$ is fuzzy-convex i.e., $\mu_{\tilde{A}}(\lambda \cdot x_1 + (1 - \lambda) \cdot x_2) \ge \max\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\} \forall x_1, x_2 \in X$, $\lambda \in [0, 1]$;*
(3) *The non-membership function $\nu_{\tilde{A}}$ is fuzzy-concave i.e. $\nu_{\tilde{A}}(\lambda \cdot x_1 + (1 - \lambda) \cdot x_2) \le \max\{\nu_{\tilde{A}}(x_1), \nu_{\tilde{A}}(x_2)\} \forall x_1, x_2 \in X, \lambda \in [0, 1]$;*
(4) *The membership and the non-membership function of $\tilde{A}$ satisfying the condition $0 \le f_1(x) + g_1(x) \le 1$, $0 \le f_2(x) + g_2(x) \le 1$ have the following form:*

$$\mu_{\tilde{A}}(x) = \begin{cases} f_1(x), & for \; a_1 \le x \le a_2, \\ 1, & for \; a_2 \le x \le a_3, \\ f_2(x), & for \; a_3 \le x \le a_4, \\ 0, & otherwise . \end{cases} \tag{9}$$

and

$$\nu_{\tilde{A}}(x) = \begin{cases} g_1(x), & for \; a_1^{'} \le x \le a_2, \\ 0, & for \; a_2 \le x \le a_3, \\ g_2(x), & for \; a_3 \le x \le a_4^{'} \\ 1, & otherwise . \end{cases} \tag{10}$$

respectively, where $f_1(x)$ and $f_2(x)$ are strictly increasing and decreasing functions in $[a_1, a_2]$ and $[a_3, a_4]$; and $g_1(x)$ and $g_2(x)$ are strictly decreasing and increasing functions in $[a_1^{'}, a_2]$ and $[a_3, a_4^{'}]$, respectively. Symbolically the intuitionistic fuzzy number is represented as

$$\tilde{A}_{IFN} = \left(a_1, a_2, a_3, a_4; a_1^{'}, a_2, a_3, a_4^{'}\right).$$

The α-cut of Intuitionistic Fuzzy Number $\tilde{A}_{IFN} = \left(a_1, a_2, a_3, a_4; a_1^{'}, a_2, a_3, a_4^{'}\right)$ generates the following pair of intervals:

$$[\tilde{A}_{IFN}]_\alpha = \left\{\left[\tilde{A}_\mu^L(\alpha), \tilde{A}_\mu^R(\alpha)\right]; \left[\tilde{A}_\nu^L(\alpha), \tilde{A}_\nu^R(\alpha)\right]\right\},$$

where the interval $\left[\tilde{A}_\mu^L(\alpha), \tilde{A}_\mu^R(\alpha)\right]$ can be defined as follows:

$$\begin{aligned} \tilde{A}_\mu^L(\alpha) &= \begin{cases} \inf\{x | \mu_{\tilde{A}}(x) \ge \alpha\}, & \text{if } \alpha > 0, \\ \inf\{x | x \in [a_2, a_3]\}, & \text{if } \alpha = 0, \end{cases} \\ \tilde{A}_\mu^R(\alpha) &= \begin{cases} \sup\{x | \mu_{\tilde{A}}(x) \ge \alpha\}, & \text{if } \alpha > 0, \\ \sup\{x | x \in [a_2, a_3]\}, & \text{if } \alpha = 0. \end{cases} \end{aligned} \tag{11}$$

In similar manner, the interval $\left[\tilde{A}_\nu^L(\alpha), \tilde{A}_\nu^R(\alpha)\right]$ can be defined as follows:

$$\begin{aligned} \tilde{A}_\nu^L(\alpha) &= \begin{cases} \inf\{x | \nu_{\tilde{A}}(x) \le 1 - \alpha\}, & \text{if } \alpha > 0, \\ \inf\{x | x \in [a_1^{'}, a_4^{'}]\}, & \text{if } 1 - \alpha = 0, \end{cases} \\ \tilde{A}_\nu^R(\alpha) &= \begin{cases} \sup\{x | \nu_{\tilde{A}}(x) \le 1 - \alpha\}, & \text{if } 1 - \alpha > 0, \\ \sup\{x | x \in [a_1^{'}, a_4^{'}]\}, & \text{if } 1 - \alpha = 0. \end{cases} \end{aligned} \tag{12}$$

DEFINITION 5. *An Intuitionistic Fuzzy Number $\tilde{A}_{IFN} = \left(a_1, a_2, a_3, a_4; a_1^{'}, a_2, a_3, a_4^{'}\right)$ defined on the universal set R is said to be a trapezoidal intuitionistic fuzzy number if and only if its membership and non-membership functions have the following form:*

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & for \; a_1 \le x \le a_2, \\ 1, & for \; a_2 \le x \le a_3, \\ \frac{a_4 - x}{a_4 - a_3}, & for \; a_3 \le x \le a_4, \\ 0, & otherwise. \end{cases} \tag{13}$$

and

$$\nu_{\tilde{A}}(x) = \begin{cases} \frac{a_2 - x}{a_2 - a_1^{'}}, & for \; a_1^{'} \le x \le a_2, \\ 0, & for \; a_2 \le x \le a_3, \\ \frac{x - a_3}{a_4^{'} - a_3}, & for \; a_3 \le x \le a_4^{'}, \\ 1, & otherwise. \end{cases} \tag{14}$$

International Journal of Applied Information Systems (IJAIS) ISSN : 2249-0868
Foundation of Computer Science FCS, New York, USA
Volume 7 - No. 7, August 2014 - www.ijais.org

where $a_1^{'} \leq a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_4^{'}$ *and* $\mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) \leq 0.5$ *for* $\mu_{\tilde{A}}(x) = \nu_{\tilde{A}}(x)\ \forall x \in R$.

Using the equations (11) and (12), the α-cuts of $\tilde{A}_{TPIFN}$ can be defined as

$$[\tilde{A}_{TPIFN}]_\alpha = \left\{ \left[\tilde{A}_\mu^L(\alpha), \tilde{A}_\mu^R(\alpha)\right]; \left[\tilde{A}_\nu^L(\alpha), \tilde{A}_\nu^R(\alpha)\right] \right\}$$
$$= \left\{ [a_1 + \alpha(a_2 - a_1), a_4 - \alpha(a_4 - a_3)];\ \left[a_2 - \alpha(a_2 - a_1^{'}), a_3 + \alpha(a_4^{'} - a_3)\right] \right\} \quad (15)$$

3.1 Arithmetic Operations on Intuitionistic Fuzzy Numbers

The arithmetic operation $(*)$ of two intuitionistic fuzzy numbers is a mapping from cartesian product space $R \times R$ onto R. If $\tilde{A}_{IFN}$ and $\tilde{B}_{IFN}$ be two IFNs then their resultant is also an IFN. Using the extension principle, the membership and the non-membership function of the resultant IFN $\tilde{A}_{IFN} * \tilde{B}_{IFN}$ can be defined as follows:

$$\mu_{\tilde{A}_{IFN}*\tilde{B}_{IFN}}(z) = \sup_{z=x*y} \min\left\{\mu_{\tilde{A}_{IFN}}(x), \mu_{\tilde{B}_{IFN}}(y)\right\} \quad (16)$$

$$\nu_{\tilde{A}_{IFN}*\tilde{B}_{IFN}}(z) = \inf_{z=x*y} \max\left\{\nu_{\tilde{A}_{IFN}}(x), \nu_{\tilde{B}_{IFN}}(y)\right\}, \quad (17)$$

where $*$ stands for any of the four arithmetic operations (addition, subtraction, multiplication, division).

3.1.1 Arithmetic Operation of Intuitionistic Fuzzy Number based on α- cut method. Using the α-cut method, arithmetic operations on intuitionistic fuzzy numbers can be defined as follows.

Let $\tilde{A}_{IFN} = \left(a_1, a_2, a_3, a_4; a_1^{'}, a_2, a_3, a_4^{'}\right)$ and $\tilde{B}_{IFN} = \left(b_1, b_2, b_3, b_4; b_1^{'}, b_2, b_3, b_4^{'}\right)$, be two IFNs, then the α-cuts of $\tilde{A}_{TPIFN}$ and $\tilde{B}_{TPIFN}$ are defined as

$$[\tilde{A}_{TPIFN}]_\alpha = \left\{ \left[\tilde{A}_\mu^L(\alpha), \tilde{A}_\mu^R(\alpha)\right]; \left[\tilde{A}_\nu^L(\alpha), \tilde{A}_\nu^R(\alpha)\right] \right\}$$
$$= \left\{ [a_1 + \alpha(a_2 - a_1), a_4 - \alpha(a_4 - a_3)];\ \left[a_2 - \alpha(a_2 - a_1^{'}), a_3 + \alpha(a_4^{'} - a_3)\right] \right\},$$

and

$$[\tilde{B}_{TPIFN}]_\alpha = \left\{ \left[\tilde{B}_\mu^L(\alpha), \tilde{B}_\mu^R(\alpha)\right]; \left[\tilde{B}_\nu^L(\alpha), \tilde{B}_\nu^R(\alpha)\right] \right\}$$
$$= \left\{ [b_1 + \alpha(b_2 - b_1), b_4 - \alpha(b_4 - b_3)];\ \left[b_2 - \alpha(b_2 - b_1^{'}), b_3 + \alpha(b_4^{'} - b_3)\right] \right\}$$

respectively.

(1) **Addition of two TPIFNs:** The addition of $\tilde{A}_{TPIFN}$ and $\tilde{B}_{TPIFN}$ is defined as follows: $[\tilde{A}_{TPIFN}]_\alpha \oplus [\tilde{B}_{TPIFN}]_\alpha$
$$= \left\{ \left[\tilde{A}_\mu^L(\alpha) + \tilde{B}_\mu^L(\alpha), \tilde{A}_\mu^R(\alpha) + \tilde{B}_\mu^R(\alpha)\right]; \left[\tilde{A}_\nu^L(\alpha) + \tilde{B}_\nu^L(\alpha), \tilde{A}_\nu^R(\alpha) + \tilde{B}_\nu^R(\alpha)\right] \right\}$$
and their membership and the non-membership functions are of the form:

$$\mu_{\tilde{A}_{TPIFN}\oplus\tilde{B}_{TPIFN}}(x) = \begin{cases} \frac{x-(a_1+b_1)}{(a_2+b_2)-(a_1+b_1)}, & \text{for } a_1 + b_1 \leq x \leq a_2 + b_2, \\ 1, & \text{for } a_2 + b_2 \leq x \leq a_3 + b_3, \\ \frac{(a_4+b_4)-x}{(a_4+b_4)-(a_3+b_3)}, & \text{for } a_3 + b_3 \leq x \leq a_4 + b_4, \\ 0, & \text{otherwise.} \end{cases}$$

and

$$\nu_{\tilde{A}_{TPIFN}\oplus\tilde{B}_{TPIFN}}(x) = \begin{cases} \frac{(a_2+b_2)-x}{(a_2+b_2)-(a_1^{'}+b_1^{'})}, & \text{for } a_1^{'} + b_1^{'} \leq x \leq a_2 + b_2, \\ 0, & \text{for } a_2 + b_2 \leq x \leq a_3 + a_3, \\ \frac{x-(a_3+b_3)}{(a_4^{'}+b_4^{'})-(a_3+b_3)}, & \text{for } a_3 + a_3 \leq x \leq a_4^{'} + b_4^{'}, \\ 1, & \text{otherwise.} \end{cases}$$

respectively. It may be observe that the addition of two TPIFNs is also a TPIFN.

(2) **Subtraction of Two TPIFNs:**
The subtraction of $\tilde{A}_{TPIFN}$ and $\tilde{B}_{TPIFN}$ is defined as follows: $[\tilde{A}_{TPIFN}]_\alpha - [\tilde{B}_{TPIFN}]_\alpha$
$$= \left\{ \left[\tilde{A}_\mu^L(\alpha) - \tilde{B}_\mu^R(\alpha), \tilde{A}_\mu^R(\alpha) - \tilde{B}_\mu^L(\alpha)\right]; \left[\tilde{A}_\nu^L(\alpha) - \tilde{B}_\nu^R(\alpha), \tilde{A}_\nu^R(\alpha) - \tilde{B}_\nu^L(\alpha)\right] \right\}$$
and their membership and non-membership functions are of the form:

$$\mu_{\tilde{A}_{TPIFN}\oplus\tilde{B}_{TPIFN}}(x) = \begin{cases} \frac{x-(a_1-b_4)}{(a_2-b_3)-(a_1-b_4)} & for\ a_1 - b_4 \leq x \leq a_2 - b_3, \\ 1 & for\ a_2 - b_3 \leq x \leq a_3 - b_2, \\ \frac{(a_4-b_1)-x}{(a_4-b_1)-(a_3-b_2)} & for\ a_3 - b_2 \leq x \leq a_4 - b_1, \\ 0 & otherwise \end{cases}$$

and

$$\nu_{\tilde{A}_{TPIFN}\oplus\tilde{B}_{TPIFN}}(x) = \begin{cases} \frac{(a_2-b_3)-x}{(a_2-b_3)-(a_1^{'}-b_4^{'})} & for\ a_1^{'} - b_4^{'} \leq x \leq a_2 - b_3, \\ 0, & for\ a_2 - b_3 \leq x \leq a_3 - b_2, \\ \frac{x-(a_3-b_2)}{(a_4^{'}-b_1^{'})-(a_3-b_2)} & for\ a_3 - b_2 \leq x \leq a_4^{'} - b_1^{'}, \\ 1, & otherwise, \end{cases}$$

respectively. It may be observe that the subtraction of two TPIFNs is also a TPIFN.

(3) **Multiplication of two TPIFNs:** The multiplication of two trapezoidal intuitionistic fuzzy numbers $\tilde{A}_{TPIFN}$ and $\tilde{B}_{TPIFN}$ is defined as follows: $[\tilde{A}_{TPIFN}]_\alpha \otimes [\tilde{B}_{TPIFN}]_\alpha$
$$= \left\{ \left[\tilde{A}_\mu^L(\alpha) \cdot \tilde{B}_\mu^L(\alpha), \tilde{A}_\mu^R(\alpha) \cdot \tilde{B}_\mu^R(\alpha)\right]; \left[\tilde{A}_\nu^L(\alpha) \cdot \tilde{B}_\nu^L(\alpha), \tilde{A}_\nu^R(\alpha) \cdot \tilde{B}_\nu^R(\alpha)\right] \right\}$$
Let us assume that

$$\begin{aligned} z_1 &= (a_1 + (a_2 - a_1)\alpha) \cdot (b_1 + (b_2 - b_1)\alpha) \\ &= (a_2 - a_1)(b_2 - b_1)\alpha^2 + (a_1 b_2 - 2a_1 b_1 + b_1 a_2)\,\alpha + a_1 b_1, \end{aligned} \quad (18)$$

$$\begin{aligned} z_2 &= (a_4 - \alpha(a_4 - a_3)\alpha) \cdot (b_4 - \alpha(b_4 - b_3)\alpha) \\ &= (a_4 - a_3)(b_4 - b_3)\alpha^2 + (a_4 b_3 - 2a_4 b_4 + a_3 b_4)\,\alpha + a_4 b_4, \end{aligned} \quad (19)$$

$$\begin{aligned} z_3 &= \left(a_2 - (a_2 - a_1^{'})\alpha\right) \cdot \left(b_2 - (b_2 - b_1^{'})\alpha\right) \\ &= (a_2 - a_1^{'})(b_2 - b_1^{'})\alpha^2 + \left(a_1^{'} b_2 - 2a_2 b_2 + a_2 b_1^{'}\right)\alpha + a_2 b_2, \end{aligned} \quad (20)$$

and

$$\begin{aligned} z_4 &= \left(a_3 + (a_4^{'} - a_3)\alpha\right) \cdot \left(b_3 + (b_4^{'} - b_3)\alpha\right) \\ &= (a_4^{'} - a_3)(b_4^{'} - b_3)\alpha^2 + \left(a_4^{'} b_3 - 2a_3 b_3 + a_3 b_4^{'}\right)\alpha + a_3 b_3. \end{aligned} \quad (21)$$

Solving the equations (18), (19), (20) and (21) for the variable α, we get

$$\alpha = \frac{-B_1 + \sqrt{B_1^2 - 4A_1(a_1 b_1 - x)}}{2A_1}, \quad (22)$$

$$\alpha = \frac{-B_1^{'} + \sqrt{{B_1^{'}}^2 - 4A_1^{'}(a_2 b_2 - x)}}{2A_1^{'}}, \quad (23)$$

$$\alpha = \frac{-B_1^{'} + \sqrt{{B_1^{'}}^2 - 4A_1^{'}(a_2 b_2 - x)}}{2A_1^{'}}, \quad (24)$$

and

$$\alpha = \frac{-B_1^{'} + \sqrt{{B_1^{'}}^2 - 4A_1^{'}(a_2 b_2 - x)}}{2A_1^{'}} \quad (25)$$

respectively, where

$$\begin{aligned} A_1 &= (a_2 - a_1)(b_2 - b_1), \\ B_1 &= (a_1 b_2 - 2a_1 b_1 + b_1 a_2)\,, \\ A_2 &= (a_4 - a_3)(b_4 - b_3), \\ B_2 &= (a_4 b_3 - 2a_4 b_4 + a_3 b_4) \\ A_1^{'} &= (a_2 - a_1^{'})(b_2 - b_1^{'}), \\ B_1^{'} &= \left(a_1^{'} b_2 - 2a_2 b_2 + a_2 b_1^{'}\right), \\ A_2^{'} &= (a_4^{'} - a_3)(b_4^{'} - b_3), \\ B_2^{'} &= \left(a_4^{'} b_3 - 2a_3 b_3 + a_3 b_4^{'}\right). \end{aligned}$$

International Journal of Applied Information Systems (IJAIS) ISSN : 2249-0868
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Volume 7 - No. 7, August 2014 - www.ijais.org

For the sake of simplicity, it may be assumed that z_1, z_2, z_3 and z_4 are the dummy variables for the variable x. Therefore, using the equations (22) and (23) and the membership function is given below:

$$\mu_{\tilde{A}\oplus\tilde{B}}(x) = \begin{cases} \frac{-B_1+\sqrt{B_1^2-4A_1(a_1b_1-x)}}{2A_1}, & \text{if } a_1b_1 \le x \le a_2b_2, \\ 1, & \text{if } a_2b_2 \le x \le a_3b_3, \\ \frac{-B_2-\sqrt{B_2^2-4A_2(a_4b_4-x)}}{2A_2}, & \text{if } a_3b_3 \le x \le a_4b_4, \\ 0, & \text{otherwise.} \end{cases}$$

Similarly, using the equations (24) and (25), we write the non-membership function in a compact form:

$$\nu_{\tilde{A}\oplus\tilde{B}}(x) = \begin{cases} \frac{-B_1^{'}+\sqrt{B_1^{'2}-4A_1^{'}(a_2b_2-x)}}{2A_1^{'}}, & \text{if } a_1^{'}b_1^{'} \le x \le a_2b_2, \\ 0, & \text{if } a_2b_2 \le x \le a_3b_3, \\ \frac{-B_2^{'}-\sqrt{B_2^{'2}-4A_2^{'}(a_3b_3-x)}}{2A_2^{'}}, & \text{if } a_3b_3 \le x \le a_4^{'}b_4^{'}, \\ 1, & \text{otherwise.} \end{cases}$$

It may be noted that the multiplication of two TPIFNs is not a TPIFN.

Example 1: Let $\tilde{A}_1 = (1,\, 2,\, 3,\, 4;\, 0.5,\, 2,\, 3,\, 4.5)$ and $\tilde{A}_2 = (2,\, 3,\, 4,\, 5;\, 1,\, 3,\, 4,\, 5.5)$ be two TPIFNs. Then the membership and the non-membership functions of their product $\tilde{A}_1 \otimes \tilde{A}_2 = (2,\, 6,\, 12,\, 20;\, 0.5,\, 6,\, 12,\, 24.75)$ using the α-cut method is defined as follows:

$$\mu_{\tilde{A}_1\otimes\tilde{A}_2}(x) = \begin{cases} \frac{-3+\sqrt{9-4(2-x)}}{2}, & \text{if } 2 \le x \le 6, \\ 1, & \text{if } 6 \le x \le 12, \\ \frac{9-\sqrt{81-4(20-x)}}{2}, & \text{if } 12 \le x \le 20, \\ 0, & \text{otherwise.} \end{cases} \tag{26}$$

and

$$\nu_{\tilde{A}_1\otimes\tilde{A}_2}(x) = \begin{cases} \frac{8.5-\sqrt{72.25-12(6-x)}}{6}, & \text{if } 0.5 \le x \le 6, \\ 0, & \text{if } 6 \le x \le 12, \\ \frac{-10.5+\sqrt{110.25-9(12-x)}}{4.5}, & \text{if } 12 \le x \le 24.75, \\ 1, & \text{otherwise,} \end{cases} \tag{27}$$

respectively.
Above $\tilde{A}_1 \otimes \tilde{A}_2$ is a trapezoidal shaped intuitionistic fuzzy number. It can also be approximated to TPIFN as $\tilde{A}_1 \otimes \tilde{A}_2 = (2,\, 6,\, 12,\, 20;\, 0.5,\, 6,\, 12,\, 24.75)$ with membership and non-membership functions as follows:

$$\mu_{\tilde{A}_1\otimes\tilde{A}_2}(x) = \begin{cases} \frac{x-2}{4}, & \text{if } 2 \le x \le 6, \\ 1, & \text{if } 6 \le x \le 12, \\ \frac{20-x}{8}, & \text{if } 12 \le x \le 20, \\ 0, & \text{otherwise.} \end{cases} \tag{28}$$

and

$$\nu_{\tilde{A}_1\otimes\tilde{A}_2}(x) = \begin{cases} \frac{6-x}{5.5}, & \text{if } 0.5 \le 6, \\ 0, & \text{if } 6 \le x \le 12, \\ \frac{x-12}{12.75}, & \text{if } 12 \le x \le 24.75, \\ 1, & \text{otherwise.} \end{cases} \tag{29}$$

respectively.
It may be observed that the membership and non-membership functions will be in the exact shape if the arithmetic operations on TPIFNs using the α-cut method are used. Therefore, it is suggested that the reliability evaluation process of a system is more accurate with the proposed method.

4. INTUITIONISTIC FUZZY RELIABILITY OF K-OUT-OF-N SYSTEM

To analyze the reliability of the system the most important consideration is that the reliability values of the components are mostly obtained by statistical data in the past or based on the subjective judgemental of the experts's experience. The data have uncertainty itself because it is extracted from various sources such as historical records, reliability databases, and system reliability experts opinion. Thus, the uncertainty in the values is an undeniable fact and it is necessary to define fuzzy value in probabilistic space and possibility of failure instead of failure probability. In this work, the uncertainty of the values of reliability of each component is represented by a trapezoidal intuitionistic fuzzy number. In this section, the intuitionistic fuzzy reliability of a k-out-of-n system with independent components is evaluated, where the intuitionistic fuzzy reliabilities of the components are not necessarily identical.
Consider a system of n independent and non-identical components with unknown reliability $R_i,\ i = 1,\, 2, \ldots,\, n$ and suppose that for each component we observe m trapezoidal intuitionistic fuzzy numbers for the reliability of R_i. These trapezoidal intuitionistic fuzzy numbers are indicated by $R_{ij},\ j = 1,\, 2,\, \ldots,\, m$, where $R_{ij} = \left(r_{ij}^1,\, r_{ij}^2,\, r_{ij}^3,\, r_{ij}^4;\, r_{ij}^{1'},\, r_{ij}^2,\, r_{ij}^3,\, r_{ij}^{4'}\right)$. For each component calculated intuitionistic fuzzy arithmetic mean of R_i is as follows:

$$\begin{aligned}\bar{R}_i &= \left(\bar{r}_i^1,\, \bar{r}_i^2,\, \bar{r}_i^3,\, \bar{r}_i^4; \bar{r}_i^{1'},\, \bar{r}_i^2,\, \bar{r}_i^3,\, \bar{r}_i^{4'}\right) \\ &= \Bigg(\frac{1}{m}\sum_{j=1}^{m} r_{ij}^1,\, \frac{1}{m}\sum_{j=1}^{m} r_{ij}^2,\, \frac{1}{m}\sum_{j=1}^{m} r_{ij}^3,\, \frac{1}{m}\sum_{j=1}^{m} r_{ij}^4;\, \frac{1}{m}\sum_{j=1}^{m} r_{ij}^{1'}, \\ &\quad \frac{1}{m}\sum_{j=1}^{m} r_{ij}^2,\, \frac{1}{m}\sum_{j=1}^{m} r_{ij}^3,\, \frac{1}{m}\sum_{j=1}^{m} r_{ij}^{4'}\Bigg),\, i = 1,\, 2,\, \ldots,\, n.\end{aligned}$$

Taking the α-cuts of each $\bar{R}_i,\ i = 1,\, 2,\, \ldots,\, n$, which is given by

$$\begin{aligned}[\bar{R}_i]_\alpha &= \left\{\left[\bar{R}_\mu^{L_i}(\alpha), \bar{R}_\mu^{U_i}(\alpha)\right]; \left[\bar{R}_\nu^{L_i}(\alpha), \bar{R}_\nu^{U_i}(\alpha)\right]\right\} \\ &= \left\{[\bar{r}_i^1 + (\bar{r}_i^2 - \bar{r}_i^1)\alpha,\ \bar{r}_i^4 + (\bar{r}_i^3 - \bar{r}_i^4)\alpha]\ ; \right. \\ &\quad \left. \left[\bar{r}_i^2 - (\bar{r}_i^2 - \bar{r}_i^{1'})\alpha,\ \bar{r}_i^3 + (\bar{r}_i^{4'} - \bar{r}_i^3)\alpha\right]\right\}.\end{aligned}$$

Now, transfer these α-cut intervals into statistical confidence intervals:
Case 1: When the population standard deviation σ is known.
In this case, the α-cuts of $(1-\gamma)100\%$ confidence interval of $\bar{R}_i$ is given by

$$[\hat{R}_i]_\alpha = \left\{\left[R_\mu^{L_i^*}(\alpha), R_\mu^{U_i^*}(\alpha)\right]; \left[R_\nu^{L_i^*}(\alpha), R_\nu^{U_i^*}(\alpha)\right]\right\}$$

Here

$$R_\mu^{L_i^*}(\alpha) = \bar{r}_i^1 + (\bar{r}_i^2 - \bar{r}_i^1)\alpha - \frac{\sigma}{\sqrt{m}} t_{m-1}(\gamma_1),$$

$$R_\mu^{U_i}(\alpha) = \bar{r}_i^4 + (\bar{r}_i^3 - \bar{r}_i^4)\alpha + \frac{\sigma}{\sqrt{m}} t_{m-1}(\gamma_2),$$

$$R_\nu^{L_i^*}(\alpha) = \bar{r}_i^2 - (\bar{r}_i^2 - \bar{r}_i^{1'})\alpha - \frac{\sigma}{\sqrt{m}} t_{m-1}(\gamma_1),$$

$$R_\nu^{U_i^*}(\alpha) = \bar{r}_i^3 + (\bar{r}_i^{4'} - \bar{r}_i^3)\alpha + \frac{\sigma}{\sqrt{m}} t_{m-1}(\gamma_2).$$

Case 2: When the population standard deviation σ is not known.
In this case, the α-cuts of $(1-\gamma)100\%$ confidence interval of $\bar{R}_i$ is given by

$$[\hat{R}_i]_\alpha = \left\{\left[R_\mu^{L_i^*}(\alpha), R_\mu^{U_i^*}(\alpha)\right]; \left[R_\nu^{L_i^*}(\alpha), R_\nu^{U_i^*}(\alpha)\right]\right\}$$

Here

$$R_\mu^{L_i^*}(\alpha) = \bar{r}_i^1 + (\bar{r}_i^2 - \bar{r}_i^1)\alpha - \frac{s_i}{\sqrt{m}} t_{m-1}(\gamma_1),$$

International Journal of Applied Information Systems (IJAIS) ISSN : 2249-0868
Foundation of Computer Science FCS, New York, USA
Volume 7 - No. 7, August 2014 - www.ijais.org

$$R_{\mu}^{U_i}(\alpha) = \bar{r}_i^4 + (\bar{r}_i^3 - \bar{r}_i^4)\alpha + \frac{s_i}{\sqrt{m}} t_{m-1}(\gamma_2),$$

$$R_{\nu}^{L_i^*}(\alpha) = \bar{r}_i^2 - (\bar{r}_i^2 - \bar{r}_i^{1'})\alpha - \frac{s_i}{\sqrt{m}} t_{m-1}(\gamma_1),$$

$$R_{\nu}^{U_i^*}(\alpha) = \bar{r}_i^3 + (\bar{r}_i^{4'} - \bar{r}_i^3)\alpha + \frac{s_i}{\sqrt{m}} t_{m-1}(\gamma_2),$$

where $\gamma_1 + \gamma_2 = \gamma$, $0 < \gamma_1,\ \gamma_2,\ \gamma < 1$.
Let T be a t-distributed random variable with $m_i - 1$ degree of freedom. Then $t_{m_i-1}(\gamma_k)$ satisfies the condition
$p(T \geq t_{m_i-1}(\gamma_k)) = \gamma_k$, $k = 1,\ 2$.
The decision maker not only chooses γ_1 and γ_2 to satisfy the condition $\gamma_1 + \gamma_2 = \gamma$, $0 < \gamma_1,\ \gamma_2,\ \gamma < 1$, but also satisfies the following conditions:

$$0 < \bar{r}_i^1 + (\bar{r}_i^2 - \bar{r}_i^1)\alpha - \frac{\sigma}{\sqrt{m}} t_{m-1}(\gamma_1) < 1, \tag{30}$$

$$0 < \bar{r}_i^4 + (\bar{r}_i^3 - \bar{r}_i^4)\alpha + \frac{\sigma}{\sqrt{m}} t_{m-1}(\gamma_2) < 1, \tag{31}$$

$$0 < \bar{r}_i^2 - (\bar{r}_i^2 - \bar{r}_i^{1'})\alpha - \frac{\sigma}{\sqrt{m}} t_{m-1}(\gamma_1) < 1, \tag{32}$$

$$0 < \bar{r}_i^3 + (\bar{r}_i^{4'} - \bar{r}_i^3)\alpha + \frac{\sigma}{\sqrt{m}} t_{m-1}(\gamma_2) < 1, \tag{33}$$

$$0 < \bar{r}_i^1 + (\bar{r}_i^2 - \bar{r}_i^1)\alpha - \frac{s_i(\alpha)}{\sqrt{m}} t_{m-1}(\gamma_1) < 1, \tag{34}$$

$$0 < \bar{r}_i^4 + (\bar{r}_i^3 - \bar{r}_i^4)\alpha + \frac{s_i(\alpha)}{\sqrt{m}} t_{m-1}(\gamma_2) < 1, \tag{35}$$

$$0 < \bar{r}_i^2 - (\bar{r}_i^2 - \bar{r}_i^{1'})\alpha - \frac{s_i(\alpha)}{\sqrt{m}} t_{m-1}(\gamma_1) < 1, \tag{36}$$

and

$$0 < \bar{r}_i^3 + (\bar{r}_i^{4'} - \bar{r}_i^3)\alpha + \frac{s_i(\alpha)}{\sqrt{m}} t_{m-1}(\gamma_2) < 1, \tag{37}$$

where $i = 1,\ 2, \ldots,\ m$.
In order to calculate the estimated value of the variance between TPIFNs, the distance formula between two IFNs is used [24]. Hence, the variance between m TPIFNs is given by

$$s_i^2(\alpha) = \frac{1}{4(m-1)} \int_0^1 \left[\sum_{j=1}^{m} \left(R_{\mu}^{L_j}(\alpha) - \bar{R}_{\mu}^{L_i}(\alpha) \right)^2 + \left(R_{\mu}^{U_j}(\alpha) - \bar{R}_{\mu}^{U_i}(\alpha) \right)^2 + \left(R_{\nu}^{L_j}(\alpha) - \bar{R}_{\nu}^{L_i}(\alpha) \right)^2 + \left(R_{\nu}^{U_j}(\alpha) - \bar{R}_{\nu}^{U_i}(\alpha) \right)^2 \right] d\alpha.$$

Ultimately, evaluating the intuitionistic fuzzy reliability of k-out-of-n system using the concept of minimal path sets, the intuitionistic fuzzy reliability of any system is equal to the possibility that at least one of the minimal path set works. A minimal path set works, if all the components in the set work. In a k-out-of-n system, there are n_{C_k} minimal path sets. Each minimal path set contains exactly k different components. Therefore, [18] the explicit formula to evaluate the intuitionistic fuzzy reliability of k-out-of-n system using minimal path sets is as follows:

$$Rel(k,n) = \left[1 - \prod_{j=1}^{n_{C_k}} \left[1 - \prod_{i \in P_j} \left[\bar{R}_{\mu}^{L_i}(\alpha), \bar{R}_{\mu}^{U_i}(\alpha) \right] \right]; \quad 1 - \prod_{j=1}^{n_{C_k}} \left[1 - \prod_{i \in P_j} \left[\bar{R}_{\nu}^{L_i}(\alpha), \bar{R}_{\nu}^{U_i}(\alpha) \right] \right] \right] \tag{38}$$

where P_j is the j^{th} minimal path set.
A series system functions if and only if each of its component function. The intuitionistic fuzzy reliability R_s of series system is given by $R_s = \prod_{i=1}^{n} \hat{R}_i$
= $\hat{R}_1 \cdot \hat{R}_2 \cdot, \ldots, \cdot \hat{R}_n$

$$= \begin{bmatrix} \prod_{i=1}^{n} [r_{i1} + (r_{i2} - r_{i1})\alpha,\ r_{i4} - (r_{i4} - r_{i3})\alpha]; \\ \prod_{i=1}^{n} \left[r_{i2} - (r_{i2} - r'_{i1})\alpha,\ r_{i3} + (r'_{i4} - r_{i3})\alpha \right] \end{bmatrix}$$

$$= \begin{bmatrix} \left[\prod_{i=1}^{n} (r_{i1} + (r_{i2} - r_{i1})\alpha) \prod_{i=1}^{n} (r_{i4} - (r_{i4} - r_{i3})\alpha) \right]; \\ \left[\prod_{i=1}^{n} \left(r_{i2} - (r_{i2} - r'_{i1})\alpha \right),\ \prod_{i=1}^{n} \left(r_{i3} + (r'_{i4} - r_{i3})\alpha \right) \right] \end{bmatrix} \tag{39}$$

A parallel system functions if and only if at least one component functions. The intuitionistic fuzzy reliability R_p of parallel the system is given by

$$R_p = 1 - \prod_{i=1}^{n} (1 - \hat{R}_i)$$

$$= \begin{bmatrix} [1,\ 1] - \prod_{i=1}^{n} \left([1\ \ 1] - \left[R_{\mu}^{L_i}(\alpha), R_{\mu}^{U_i}(\alpha) \right] \right); \\ [1,\ 1] - \prod_{i=1}^{n} \left([1\ \ 1] - \left[R_{\nu}^{L_i}(\alpha), R_{\nu}^{U_i}(\alpha) \right] \right) \end{bmatrix}$$

$$= \begin{bmatrix} [1,\ 1] - \left[\prod_{i=1}^{n} \left[1 - \left(r_{i4} - (r_{i4} - r_{i3})\alpha \right) \right],\ \prod_{i=1}^{n} \left[1 - \left(r_{i1} + (r_{i2} - r_{i1})\alpha \right) \right] \right]; \\ [1,\ 1] - \left[\prod_{i=1}^{n} \left[1 - \left(r_{i3} - (r_{i3} - r'_{i4})\alpha \right) \right],\ \prod_{i=1}^{n} \left[1 - \left(r_{i2} + (r'_{i1} - r_{i2})\alpha \right) \right] \right] \end{bmatrix}$$

$$= \begin{bmatrix} \left[1 - \prod_{i=1}^{n} \left[1 - \left(r_{i1} + (r_{i2} - r_{i1})\alpha \right) \right],\ 1 - \prod_{i=1}^{n} \left[1 - \left(r_{i4} - (r_{i4} - r_{i3})\alpha \right) \right] \right]; \\ \left[1 - \prod_{i=1}^{n} \left[1 - \left(r_{i2} + (r'_{i1} - r_{i2})\alpha \right) \right],\ 1 - \prod_{i=1}^{n} \left[1 - \left(r_{i3} - (r_{i3} - r'_{i4})\alpha \right) \right] \right] \end{bmatrix} \tag{40}$$

5. NUMERICAL EXAMPLE

Consider a 2-out-of-4 system with random samples of size 4 for each component C_i, $i = 1,\ 2,\ \ldots,\ 4$. Intuitionistic fuzzy statistical sample data for different components is shown in Table 1. If σ is unknown, then the α-cuts of $(1-\gamma)100\%$ statistical confidence interval with $\gamma_1 = 0.025$, $\gamma_2 = 0.025$, $\gamma = 0.05$ for the reliability R_i, $i = 1,\ 2,\ \ldots,\ 4$ are estimated. Using these confidence intervals the reliability of 2-out-of-4 system are calculated for different values of α and results are furnished in Table 2. Further, using the Table 2 the membership and the non-membership functions are plotted in figure 1.
The fuzzy arithmetic mean of R_i, $i = 1,\ 2,\ \ldots,\ 4$ are given by

$$\bar{R}_1 = (0.14,\ 0.21,\ 0.28,\ 0.34;\ 0.13,\ 0.21,\ 0.28,\ 0.36),$$

$$\bar{R}_2 = (0.34,\ 0.43,\ 0.50,\ 0.57;\ 0.24,\ 0.43,\ 0.50,\ 0.62),$$

$$\bar{R}_3 = (0.44,\ 0.53,\ 0.60,\ 0.68;\ 0.40,\ 0.53,\ 0.60,\ 0.72),$$

$$\bar{R}_4 = (0.82,\ 0.88,\ 0.91,\ 0.93;\ 0.76,\ 0.88,\ 0.91,\ 0.95).$$

The α-cuts of $\bar{R}_i$, $i = 1,\ 2,\ \ldots,\ 4$ are given by

$$[\bar{R}_1]_\alpha = \{[0.14 + 0.07\alpha,\ 0.34 - 0.06\alpha];\ [0.21 - 0.08\alpha,\ 0.28 + 0.08\alpha]\},$$

Table 1.

C_1	C_2
(.14, .20, .26, .34; .12, .20, .26, .36)	(.30, .40, .50, .60; .20, .40, .50, .65)
(.15, .21, .27, .35; .13, .21, .27, .37)	(.35, .45, .50, .55; .25, .45, .50, .60)
(.10, .20, .28, .30; .09, .20, .28, .33)	(.33, .44, .52, .58; .24, .44, .52, .62)
(.18, .24, .30, .36; .16, .24, .30, .39)	(.36, .41, .48, .56; .26, .41, .48, .60)
C_3	C_4
(.40, .50, .60, .70; .35, .50, .60, .80)	(.82, .91, .94, .98; .72, .91, .94, .99)
(.45, .52, .65, .73; .40, .52, .65, .75)	(.81, .89, .90, .94; .71, .89, .90, .95)
(.48, .63, .65, .70; .44, .63, .65, .73)	(.86, .88, .91, .92; .84, .88, .91, .94)
(.41, .46, .48, .58; .39, .46, .48, .60)	(.80, .85, .87, .89; .78, .85, .87, .92)

$$[\bar{R}_2]_\alpha = \{[0.34 + 0.09\alpha,\ 0.57 - 0.07\alpha];\ [0.43 - 0.15\alpha,\ 0.50 + 0.12\alpha]\},$$

$$[\bar{R}_3]_\alpha = \{[0.44 + 0.09\alpha,\ 0.68 - 0.08\alpha];\ [0.53 - 0.13\alpha,\ 0.60 + 0.12\alpha]\},$$

$$[\bar{R}_4]_\alpha = \{[0.82 + 0.06\alpha,\ 0.93 - 0.02\alpha];\ [0.88 - 0.12\alpha,\ 0.91 + 0.04\alpha]\}.$$

In case, when σ is not known, the α-cuts of 95% confidence interval for $\bar{R}_i,\ i = 1, 2, \ldots, 4$ are given by

$$[\hat{R}_1]_\alpha = \{[0.10 + 0.07\alpha,\ 0.38 - 0.06\alpha];\ [0.17 - 0.08\alpha,\ 0.32 + 0.08\alpha]\},$$

$$[\hat{R}_2]_\alpha = \{[0.29 + 0.09\alpha,\ 0.62 - 0.07\alpha];\ [0.38 - 0.15\alpha,\ 0.55 + 0.12\alpha]\},$$

$$[\hat{R}_3]_\alpha = \{[0.33 + 0.09\alpha,\ 0.79 - 0.08\alpha];\ [0.42 - 0.13\alpha,\ 0.71 + 0.12\alpha]\},$$

$$[\hat{R}_4]_\alpha = \{[0.77 + 0.06\alpha,\ 0.98 - 0.02\alpha];\ [0.83 - 0.12\alpha,\ 0.96 + 0.04\alpha]\}.$$

The minimal path sets are: $P_1 = \{R_1, R_2\}$, $P_2 = \{R_1, R_3\}$, $P_3 = \{R_1, R_4\}$, $P_4 = \{R_2, R_3\}$, $P_5 = \{R_2, R_4\}$ and $P_6 = \{R_3, R_4\}$.
Now, calculate the confidence interval for the reliability of a 2-out-of-4 system for different values of $\alpha \in [0, 1]$ by using formula given by the equation (38) and result is furnished in Table 2. The membership and non-membership functions of the obtained result are shown in figure 1.

Table 2.

α	Confidence Interval of the Reliability of 2-out-of-4 System
0.0	{[0.6899, 0.9552]; [0.8312, 0.9076]}
0.1	{[0.7060, 0.9514]; [0.8126, 0.9167]}
0.2	{[0.7218, 0.9475]; [0.7928, 0.9252]}
0.3	{[0.7371, 0.9434]; [0.7720, 0.9330]}
0.4	{[0.7520, 0.9390]; [0.7501, 0.9402]}
0.5	{[0.7664, 0.9344]; [0.7273, 0.9469]}
0.6	{[0.7803, 0.9295]; [0.7034, 0.9531]}
0.7	{[0.7938, 0.9244]; [0.6787, 0.9587]}
0.8	{[0.8067, 0.9191]; [0.6531, 0.9639]}
0.9	{[0.8192, 0.9135]; [0.6266, 0.9686]}
1.0	{[0.8312, 0.9076]; [0.5995, 0.9728]}

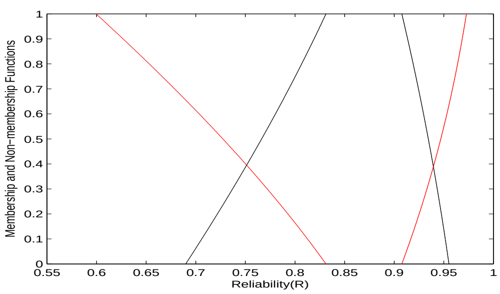

Fig. 1. *Reliability of* 2*-out-of-*4 *System*

6. CONCLUSION

The proposed new arithmetic operations on intuitionistic fuzzy numbers using the α-cut method are valid operations. As the membership and non-membership functions would be in the exact shape using the α-cut method. More accuracy with the proposed method for the reliability evaluation process of a system is acheived. A new methodology based on intuitionistic fuzzy confidence interval has been provided for analyzing the intuitionistic fuzzy system reliability of k-out-of-n system (particularly, series and parallel system) has been provided. Firstly, estimated the reliability of each component of the system using the intuitionistic fuzzy statistical sample data using the α-cuts of $(1 - \gamma)100\%$ approach and then based on the estimated reliability of the components obtained, the intuitionistic fuzzy reliability of the system has been finally calculated using the minimal path sets approach.

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