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3-Equitable Prime Cordial Labeling of Graphs

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ABSTRACT

A 3-equitable prime cordial labeling of a graph G with vertex set V is a bijection f from V to $\{1, 2, ..., |V|\}$ such that if an edge uv is assigned the label 1 if $gcd(f(u), f(v)) = 1$ and $gcd(f(u) + f(v), f(u)-f(v)) = 1$, the label 2 if $gcd(f(u), f(v)) = 1$ and $gcd(f(u) + f(v), f(u) - f(v)) = 2$ and 0 otherwise, then the number of edges labeled with i and the number of edges labeled with j differ by atmost 1 for $0 \leq i, j \leq 2$. If a graph has a 3-equitable prime cordial labeling, then it is called a 3-equitable prime cordial graph. In this paper, we investigate the 3-equitable prime cordial labeling behaviour of paths, cycles, star graphs and complete graphs.

Keywords:

3-equitable prime cordial labeling, 3-equitable prime cordial graph

1. INTRODUCTION

The graphs considered here are finite, connected, undirected and simple. The vertex set and edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. For various graph theoretic notations and terminology we follow Harary [1] and for number theory we follow Burton [2]. We will give the brief summary of definitions which are useful for the present investigations.

DEFINITION 1. *If the vertices of the graph are assigned values subject to certain conditions it is known as graph labeling.*

Most interesting graph labeling problems have three important characteristics.
1. A set of numbers from which vertex labels are chosen.
2. A rule that assigns a value to each edge.
3. A condition that these values must satisfy.
For detailed survey on graph labeling one can refer Gallian [3]. Vast amount of literature is available on different types of graph labeling. According to Beineke and Hegde [4] graph labeling serves as a frontier between number theory and structure of graphs.
Labeled graphs have variety of applications in coding theory, particularly for missile guidance codes, design of good radar type codes and convolution codes with optimal autocorrelation properties. Labeled graph plays vital role in the study of X-ray crystallography, communication network and to determine optimal circuit layouts. A detailed study of variety of applications of graph labeling is given by Bloom and Golomb [5].

The present work is to aimed to discuss one such labeling known as 3-equitable prime cordial labeling.

DEFINITION 2. *Let $G = (V(G), E(G))$ be a graph. A mapping $f : V(G) \to \{0, 1\}$ is called binary vertex labeling of G and $f(v)$ is called the label of the vertex v of G under f.*

For an edge $e = uv$, the induced edge labeling $f^* : E(G) \to \{0, 1\}$ is given by $f^*(e) = |f(u) - f(v)|$. Let $v_f(0)$, $v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f and $e_f(0)$, $e_f(1)$ be the number of edges having labels 0 and 1 respectively under f^*.

DEFINITION 3. *A binary vertex labeling of a graph G is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is cordial if it admits cordial labeling.*

The concept of cordial labeling was introduced by Cahit [6].

Many researchers have studied cordiality of graphs. Cahit [6] proved that tree is cordial and K_n is cordial if and only if $n \leq 3$. Vaidya et al. [7] have also discussed the cordiality of various graphs.

DEFINITION 4. *Let $G = (V, E)$ be a graph. A mapping $f : V(G) \to \{0, 1, 2\}$ is called ternary vertex labeling of G and $f(v)$ is called the label of the vertex v of G under f.*

For an edge $e = uv$, the induced edge labeling $f^* : E(G) \to \{0, 1, 2\}$ is given by $f^*(e) = |f(u) - f(v)|$. Let $v_f(0)$, $v_f(1)$, $v_f(2)$be the number of vertices of G having labels 0,1,2 respectively under f and $e_f(0)$, $e_f(1)$, $e_f(2)$ be the number of edges having labels 0,1,2 respectively under f^*.

DEFINITION 5. *A ternary vertex labeling of a graph G is called a 3-equitable labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $0 \leq i, j \leq 2$. A graph G is 3-equitable if it admits 3-equitable labeling.*

The concept of 3-equitable labeling was introduced by Cahit [8]. Many researchers have studied 3-equitability of graphs. Cahit [8] proved that C_n is 3-equitable except $n \equiv 3\,(mod 6)$. In the same paper he proved that an Eulerian graph with number of edges congruent to 3(mod6) is not 3-equitable. Youssef [9] proved that W_n is 3-equitable for all $n \geq 4$.

DEFINITION 6. *A prime cordial labeling of a graph G with vertex set $V(G)$ is a bijection $f : V(G) \to \{1, 2, 3, ..., |V|\}$*

defined by

$$f(e=uv) = 1 \quad if gcd(f(u), f(v)) = 1$$
$$= 0 \quad otherwise$$

and $|e_f(0) - e_f(1)| \le 1$. *A graph which admits prime cordial labeling is called a prime cordial graph.*

The concept of prime cordial labeling was introduced by Sundaram et al.[10] and in the same paper they investigate several results on prime cordial labeling. Vaidya and Vihol [11] have also discussed prime cordial labeling in the context of graph operations.

By combining the concepts of prime cordial labeling and 3-equitable labeling and by using the result in Number Theory that if gcd$(a, b) = 1$, then gcd$(a + b, a - b)$=1 or 2, we introduce a new concept called 3-equitable prime cordial labeling as follows.

DEFINITION 7. *A 3-equitable prime cordial labeling of a graph G with vertex set $V(G)$ is a bijection $f : V(G) \to \{1, 2, 3, ..., p\}$ defined by*

$$f(e=uv) = \begin{cases} 1 \quad if \quad gcd(f(u), f(v)) = 1 \\ \quad and\, gcd(f(u)+f(v), f(u)-f(v)) = 1 \\ 2 \quad if \quad gcd(f(u), f(v)) = 1 \\ \quad and\, gcd(f(u)+f(v), f(u)-f(v)) = 2 \\ 0 \quad otherwise \end{cases}$$

and $|e_f(i) - e_f(j)| \le 1$ *for all* $0 \le i, j \le 2$. *A graph which admits 3-equitable prime cordial labeling is called a 3-equitable prime cordial graph.*

2. MAIN RESULTS

Here we prove that paths P_n, cycles $C_n, n \ge 4$, stars $K_{1,n}, n \equiv 1\,(mod 3)$ and complete graphs $K_n, n \le 2$ admit 3-equitable prime cordial labeling.

THEOREM 1. *The path P_n is 3-equitable prime cordial.*

PROOF. Let $P_n : u_1 u_2 ... u_n$ be the path and let $t_i = u_i u_{i+1} (1 \le$i$\le$n-1) be the edges.
Case 1: $n \le 4$
The following table gives the 3-equitable prime cordial labeling of $P_n, n \le 4$.
Vertex labels:

n	u_1	u_2	u_3	u_4
1	1			
2	2	1		
3	2	1	3	
4	2	4	1	3

Case 2: $n \ge 5$
Let v_k be the k^{th} vertex in P_{n-1}.
Subcase 1: $n \equiv 1, 2 (mod 3)$
Define

$$u_k = v_k \quad if \quad k \le n-1$$
$$\text{and} u_n = n$$

Subcase 2: $n \equiv 0 (mod 3)$
Define

$$u_k = v_k \quad if \quad k \le n-2$$
$$u_{n-1} = n$$
$$\text{and} u_n = n-1$$

Thus in both cases we have $|e_f(i) - e_f(j)| \le 1$ for all $0 \le i, j \le 2$.
Hence P_n is 3-equitable prime cordial. □

EXAMPLE 1. *The 3-equitable prime cordial labeling of paths $P_n (5 \le$n$\le 13)$ with vertex labels and the corresponding edge labels are given in the following tables.*

Vertex labels:

n	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	u_{12}	u_{13}
5	2	4	1	3	5								
6	2	4	1	3	6	5							
7	2	4	1	3	6	5	7						
8	2	4	1	3	6	5	7	8					
9	2	4	1	3	6	5	7	9	8				
10	2	4	1	3	6	5	7	9	8	10			
11	2	4	1	3	6	5	7	9	8	10	11		
12	2	4	1	3	6	5	7	9	8	10	12	11	
13	2	4	1	3	6	5	7	9	8	10	12	11	13

Edge labels:

n	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}
2	1											
3	1	2										
4	0	1	2									
5	0	1	2	2								
6	0	1	2	0	1							
7	0	1	2	0	1	2						
8	0	1	2	0	1	2	1					
9	0	1	2	0	1	2	2	1				
10	0	1	2	0	1	2	2	1	0			
11	0	1	2	0	1	2	2	1	0	1		
12	0	1	2	0	1	2	2	1	0	0	1	
13	0	1	2	0	1	2	2	1	0	0	1	2

THEOREM 2. *The cycle C_n is 3-equitable prime cordial if and only if $n \ge 4$.*

PROOF. Let $C_n : u_1 u_2 ... u_n u_1$ be the cycle and let $t_i = u_i u_{i+1} (1 \le$i$\le$n-1) and $t_n = u_n u_1$ be the edges.
Case 1: $n = 3$
The labeling of C_3 is given as follows.

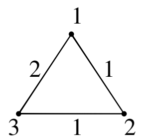

Fig 1: Labeling of C_3

We see that $|e_f(0) - e_f(1)|$=2 and hence C_3 is not 3-equitable prime cordial.
Case 2: $4 \le$n≤ 6
The following table gives the 3-equitable prime cordial labeling of $C_n, 4 \le$n≤ 6.
Vertex labels:

n	u_1	u_2	u_3	u_4	u_5	u_6
4	3	2	4	1		
5	3	5	2	4	1	
6	3	5	2	4	6	1

Case 3: $n \ge 7$
Let v_k be the k^{th} vertex in C_{n-1}.
Subcase 1: $n \equiv 1, 4, 5 (mod 6)$
Define

$$u_k = v_k \quad if \quad k \le n-1$$
$$\text{and} u_n = n$$

Subcase 2: $n \equiv 2 (mod 6)$
Let $n = 6m + 2$

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Define

$$u_k = v_k \quad if \quad k \le 2m$$
$$u_{2m+1} = v_{n-2}$$
$$u_{2m+2} = v_{n-1}$$
$$u_k = v_{k-2} \quad if \quad 2m+3 \le k \le n-1$$
$$\text{and} u_n = n$$

Subcase 3: $n \equiv 3(mod6)$
Define

$$u_k = v_k \quad if \quad k \le n-2$$
$$u_{n-1} = n$$
$$\text{and} u_n = n-1$$

Subcase 4: $n \equiv 0(mod6)$
Define

$$u_k = v_k \quad if \quad k \le n-5$$
$$u_k = k \quad if \quad n-4 \le k \le n-2$$
$$u_{n-1} = n$$
$$\text{and} u_n = n-1$$

Thus in cases(ii)and(iii) we have $|e_f(i) - e_f(j)| \le 1$ for all $0 \le i, j \le 2$.
Hence C_n is 3-equitable prime cordial if and only if $n \ge 4$.

□

EXAMPLE 2. *The 3-equitable prime cordial labeling of cycles $C_n(7 \le n \le 13)$ with vertex labels and the corresponding edge labels are given in the following tables.*

Vertex labels:

n	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	u_{12}	u_{13}
7	3	5	2	4	6	1	7						
8	3	5	1	7	2	4	6	8					
9	3	5	1	7	2	4	6	9	8				
10	3	5	1	7	2	4	6	9	8	10			
11	3	5	1	7	2	4	6	9	8	10	11		
12	3	5	1	7	2	4	6	8	9	10	12	11	
13	3	5	1	7	2	4	6	8	9	10	12	11	13

Edge labels:

n	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}	t_{13}
4	1	0	1	2									
5	2	1	0	1	2								
6	2	1	0	0	1	2							
7	2	1	0	0	1	2	2						
8	2	2	2	1	0	0	0	1					
9	2	2	2	1	0	0	0	1	1				
10	2	2	2	1	0	0	0	1	0	1			
11	2	2	2	1	0	0	0	1	0	1	2		
12	2	2	2	1	0	0	0	1	1	0	1	2	
13	2	2	2	1	0	0	0	1	1	0	1	2	2

THEOREM 3. *The star graph $K_{1,n}$ is 3-equitable prime cordial if and only if $n \equiv 2(mod3)$.*

PROOF. Let v be the central vertex and let $v_1, v_2, ..., v_n$ be the end vertices of the star $K_{1,n}$.

Case 1: $n \equiv 2(mod3)$
Assign the label 3 to the vertex v and the remaining labels to the vertices $v_1, v_2, ..., v_n$.
We see that $|e_f(i) - e_f(j)| \le 1$ for all $0 \le i, j \le 2$.
Thus $K_{1,n}$ is 3-equitable prime cordial.

Case 2: $n \equiv 0, 1(mod3)$
Subcase 1: Assign an even label to the vertex v.
There is no edge labeled 2.
Subcase 2: Assign an odd label to the vertex v.
We see that $|e_f(1) - e_f(2)|$=0 or 1 but $|e_f(0) - e_f(1)| \ge 2$ and $|e_f(0) - e_f(2)| \ge 2$.
Thus $K_{1,n}$ is not 3-equitable prime cordial.
Hence $K_{1,n}$ is 3-equitable prime cordial if and only if $n \equiv 2(mod3)$. □

EXAMPLE 3. *The 3-equitable prime cordial labeling of $K_{1,5}$ and $K_{1,8}$ are shown below.*

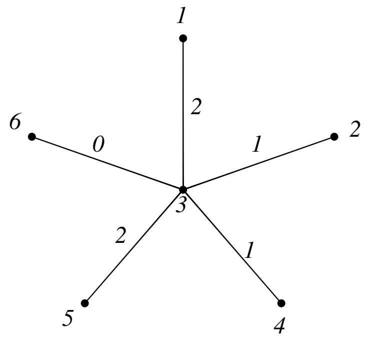

Fig 2: 3-equitable prime cordial labeling of $K_{1,5}$

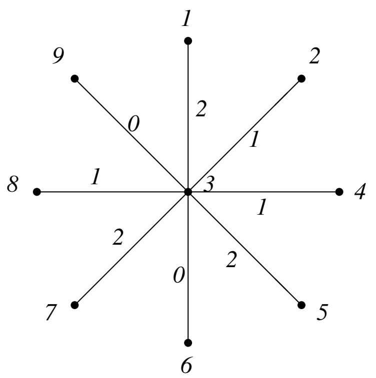

Fig 3: 3-equitable prime cordial labeling of $K_{1,8}$

THEOREM 4. *The complete graph K_n is not 3-equitable prime cordial for $n \ge 3$.*

PROOF. Let $v_1, v_2, ..., v_n$ be the vertices of K_n with $f(v_i) = i$.
Case(i): $n = 1, 2$
The 3-equitable prime cordial labeling of K_1 and K_2 are given as follows.

Fig 4: 3-equitable prime cordial labeling of K_1 and K_2

Case(ii): $n = 3$
By Fig 1, we see that $|e_f(0) - e_f(1)|$=2 and hence C_3 is not 3-equitable prime cordial.

Case(iii): $n \ge 4$
We see that $|e_f(0) - e_f(1)| \ge 3$ and hence C_3 is not 3-equitable prime cordial.
Thus K_n is not 3-equitable prime cordial for $n \ge 3$. □

3. CONCLUDING REMARKS

By using a property from number theory, we have introduced 3-equitable prime cordial labeling of graphs. In the present work, we have tried to investigate the 3-equitable prime cordial labeling behaviour of standard graphs only. To investigate analogous results for different graphs as well as in the context of various graph labeling problems is an open area of research.

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