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Centered Triangular Sum Labeling of Graphs

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ABSTRACT

A (p,q) graph G is said to admit centered triangular sum labeling if its vertices can be labeled by non negative integers such that the induced edge labels obtained by the sum of the labels of end vertices are the first q centered triangular numbers. A graph G which admits centered triangular sum labeling is called centered triangular sum graph. In this paper we prove that paths, combs, stars, subdivision of stars, bistars and coconut trees and admit centered triangular sum labeling.

Keywords:

Centered triangular numbers, centered triangular sum labelings, centered triangular sum graphs

1. INTRODUCTION

The graphs considered here are finite, connected, undirected and simple. The vertex set and edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. For various graph theoretic notations and terminology we follow Harary [1] and for number theory we follow Burton [2]. We will give the brief summary of definitions which are useful for the present investigations.

DEFINITION 1. *If the vertices of the graph are assigned values subject to certain conditions it is known as graph labeling.*

A dynamic survey on graph labeling is regularly updated by Gallian [3] and it is published by Electronic Journal of Combinatorics. Vast amount of literature is available on different types of graph labeling and more than 1000 research papers have been published so far in last four decades.
Most important labeling problems have three important ingredients.
* A set of numbers from which vertex labels are chosen;
* A rule that assigns a value to each edge;
* A condition that these values must satisfy.
The present work is to aimed to discuss one such labeling known as centered triangular sum labeling.

DEFINITION 2. *A triangular number is a number obtained by adding all positive integers less than or equal to a given positive integer n. If the n^{th} triangular number is denoted by T_n, then $T_n = \frac{1}{2}n(n+1)$.*

The triangular numbers are 1,3,6,10,15,21,28,36,45,55,...

DEFINITION 3. *A triangular sum labeling of a graph G is a one-to-one function $f : V(G) \to N$ (where N is the set of all non-negative integers) that induces a bijection $f^+ : E(G) \to \{T_1, T_2, ..., T_q\}$ of the edges of G defined by $f^+(uv) = f(u) + f(v), \forall e = uv \in E(G)$. The graph which admits such labeling is called a triangular sum graph.*

This concept was introduced by Hegde and Shankaran [4]. Motivated by the concept of triangular sum labeling, we introduce a concept called centered triangular sum labeling.

DEFINITION 4. *A centered triangular number is a centered figurative number that represents a triangle with a dot in the center and all other dots surrounding the center in successive triangular layers. If the n^{th} centered triangular number is denoted by C_n, then $C_n = \frac{1}{2}(3n^2 - 3n + 2)$.*

The following image shows the building of the centered triangular numbers using the associated figures:

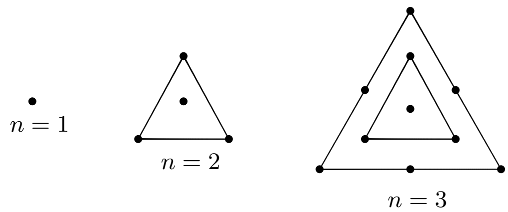

Fig 1: Building of centered triangular numbers

The first few centered triangular numbers are 1,4,10,19,31,46,64,85,109,136,166,199,235,274,...

DEFINITION 5. *A centered triangular sum labeling of a graph G is a one-to-one function $f : V(G) \to N$ that induces a bijection $f^+ : E(G) \to \{C_1, C_2, ..., C_q\}$ of the edges of G defined by $f^+(uv) = f(u) + f(v), \forall e = uv \in E(G)$. The graph which admits such labeling is called a centered triangular sum graph.*

EXAMPLE 1. *A centered triangular sum graph with 8 vertices is shown below.*

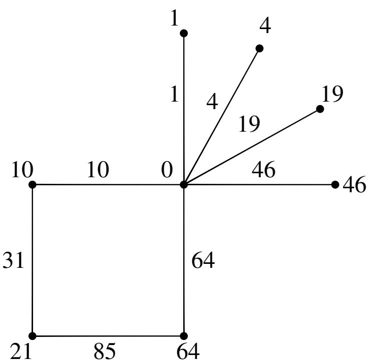

Fig 2: A centered triangular sum graph with 8 vertices

2. MAIN RESULTS

Here we prove that paths, combs, stars, subdivision of stars, bistars and coconut trees admit centered triangular sum labeling.

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THEOREM 1. *The path P_n admits centered triangular sum labeling.*

PROOF. Let $P_n : u_1u_2...u_n$ be the path and let $v_i = u_iu_{i+1}(1 \leq i \leq n-1)$ be the edges.
For i=1,2,...,n, define

$$f(u_i) = \begin{cases} \frac{3}{4}(i-1)^2 & \text{if i is odd} \\ \frac{1}{4}(3i^2 - 6i + 4) & \text{if i is even} \end{cases}$$

We will prove that the induced edge labels obtained by the sum of the labels of end vertices are the first $n-1$ centered triangular numbers.

Case(i): i is odd
For $1 \leq i \leq n-1$,

$$\begin{aligned} f(u_i) + f(u_{i+1}) &= \frac{3}{4}(i-1)^2 + \frac{1}{4}[3(i+1)^2 - 6(i+1) + 4] \\ &= \frac{1}{2}(3i^2 - 3i + 2) \\ &= C_i \\ &= f^+(v_{i)} \end{aligned}$$

Case(ii): i is even
For $1 \leq i \leq n-1$,

$$\begin{aligned} f(u_i) + f(u_{i+1}) &= \frac{1}{4}(3i^2 - 6i + 4) + \frac{3}{4}i^2 \\ &= \frac{1}{2}(3i^2 - 3i + 2) \\ &= C_i \\ &= f^+(v_{i)} \end{aligned}$$

Thus the induced edge labels are the first $n-1$ centered triangular numbers.
Hence path P_n admits centered triangular sum labeling. □

EXAMPLE 2. *The centered triangular sum labeling of P_8 is shown below.*

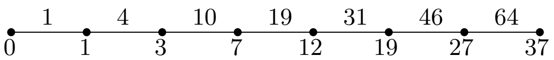

Fig 3: Centered triangular sum labeling of P_8

THEOREM 2. *The comb $P_n \odot K_1$ admits centered triangular sum labeling.*

PROOF. Let $P_n : u_1u_2...u_n$ be the path and let $v_i = u_iu_{i+1}(1 \leq i \leq n-1)$ be the edges. Let $w_1, w_2, ..., w_n$ be the pendant vertices adjacent to $u_1, u_2, ..., u_n$ respectively and $t_i = u_iw_i(1 \leq i \leq n)$ be the edges.
For i=1,2,...,n, define

$$f(u_i) = \begin{cases} \frac{3}{4}(i-1)^2 & \text{if i is odd} \\ \frac{1}{4}(3i^2 - 6i + 4) & \text{if i is even} \end{cases}$$

and

$$f(w_i) = \begin{cases} \frac{1}{4}[3i^2 + 12(n-1)i \\ +(6n^2 - 18n + 13)] & \text{if i is odd} \\ \frac{1}{4}[3i^2 + 12(n-1)i \\ +(6n^2 - 18n + 12)] & \text{if i is even.} \end{cases}$$

Then $f(u_i) + f(u_{i+1}) = f^+(v_i)$ for $1 \leq i \leq n-1$

and $f(u_i) + f(w_i) = f^+(t_i)$ for $1 \leq i \leq n$.

Thus the induced edge labels are the first $2n-1$ centered triangular numbers.
Hence comb admits centered triangular sum labeling. □

EXAMPLE 3. *The centered triangular sum labeling of $P_6 \odot K_1$ is shown below.*

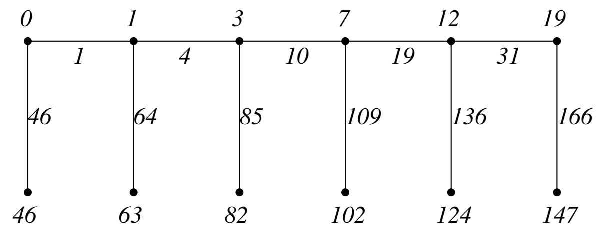

Fig 4: Centered triangular sum labeling of $P_6 \odot K_1$

THEOREM 3. *The star graph $K_{1,n}$ admits centered triangular sum labeling.*

PROOF. Let v be the apex vertex and let $v_1, v_2, ..., v_n$ be the pendant vertices of the star $K_{1,n}$.
Define

$$f(v) = 0$$
$$\text{and} f(v_i) = \frac{1}{2}(3i^2 - 3i + 2), 1 \leq i \leq n.$$

We see that the induced edge labels are the first n centered triangular numbers.
Hence $K_{1,n}$ admits centered triangular sum labeling. □

EXAMPLE 4. *The centered triangular sum labeling of $K_{1,7}$ is shown below.*

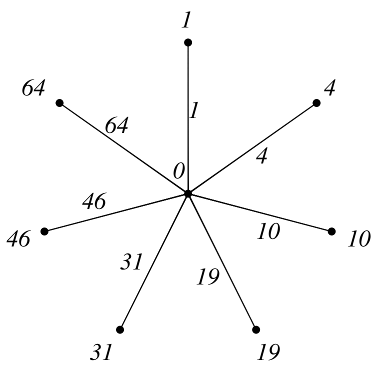

Fig 5: Centered triangular sum labeling of $K_{1,7}$

THEOREM 4. *$S(K_{1,n})$, the subdivision of the star $K_{1,n}$, admits centered triangular sum labeling.*

PROOF. Let $V(S(K_{1,n})) = \{v, v_i, u_i : 1 \leq i \leq n\}$ and $E(S(K_{1,n})) = \{vv_i, v_iu_i : 1 \leq i \leq n\}$.

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Define f by

$$f(v) = 0,$$
$$f(v_i) = \frac{1}{2}(3i^2 - 3i + 2), 1 \leq i \leq n$$
$$\text{and} f(u_i) = \frac{3}{2}[n(n-1) + 2ni], 1 \leq i \leq n.$$

We see that the induced edge labels are the first $2n$ centered triangular numbers.
Hence $S(K_{1,n})$ admits centered triangular sum labeling. □

EXAMPLE 5. *The centered triangular sum labeling of $S(K_{1,5})$ is shown below.*

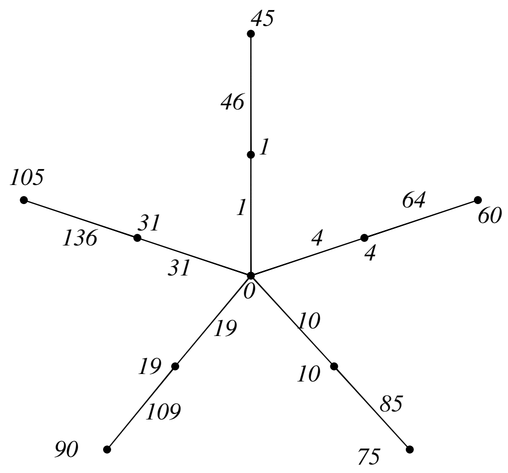

Fig 6: Centered triangular sum labeling of $S(K_{1,5})$

THEOREM 5. *The bistar $B_{m,n}$ admits centered triangular sum labeling.*

PROOF. Let $V(B_{m,n}) = \{u, v, u_i, v_j : 1 \leq i \leq m, 1 \leq j \leq n\}$ and
$E(B_{m,n}) = \{uv, uu_i, vv_j : 1 \leq i \leq m, 1 \leq j \leq n\}$.
Define f by

$$f(u) = 0,$$
$$f(v) = 1,$$
$$f(u_i) = \frac{1}{2}(3i^2 + 3i + 2), 1 \leq i \leq m$$
$$\text{and} f(v_j) = \frac{1}{2}[3j^2 + 3(2m+1)j + (3m^2 + 3m + 2)] - 1, 1 \leq j \leq n.$$

We see that the induced edge labels are the first $m + n + 1$ centered triangular numbers.
Hence $B_{m,n}$ admits centered triangular sum labeling. □

EXAMPLE 6. *The centered triangular sum labeling of $B_{4,3}$ is shown below.*

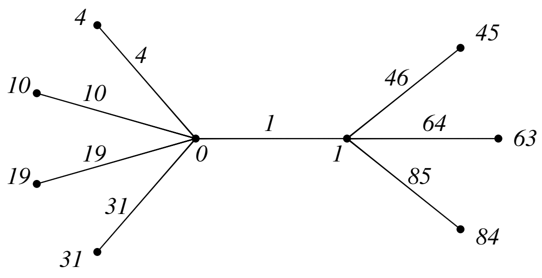

Fig 7: Centered triangular sum labeling of $B_{4,3}$

THEOREM 6. *Coconut tree admits centered triangular sum labeling.*

PROOF. Let $v_0, v_1, v_2, ..., v_i$ be the vertices of a path, having path length $i(i \geq 1)$ and $v_{i+1}, v_{i+2}, ..., v_n$ be the pendant vertices, being adjacent with v_0.
For 0≤j≤i, define

$$f(v_j) = \begin{cases} \frac{3}{4}j^2 & \text{if j is even} \\ \frac{1}{4}(3j^2 + 1) & \text{if j is odd} \end{cases}$$

and for i+1≤k≤n, define

$$f(v_k) = \frac{1}{2}(3k^2 - 3k + 2)$$

We see that the induced edge labels are the first n centered triangular numbers.
Hence coconut tree admits centered triangular sum labeling. □

EXAMPLE 7. *The centered triangular sum labeling of a coconut tree is shown below.*

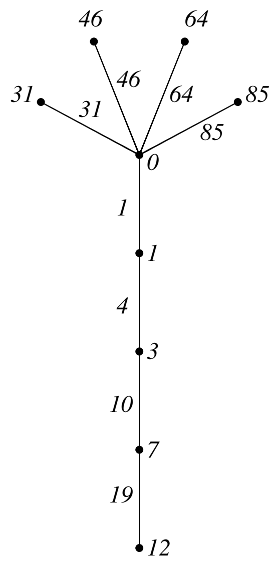

Fig 8: Centered triangular sum labeling of a coconut tree

3. CONCLUDING REMARKS

In the present work, we have tried to investigate the centered triangular sum labeling behaviour of standard graphs only. To investigate analogous results for different graphs as well as in the context of various graph labeling problems is an open area of research.

4. ACKNOWLEDGEMENT

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