



# A New Despeckling Method in Ultrasonography: Anisotropic Diffusion Filtering Followed by Total Variation Denoising

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## ABSTRACT

This paper proposes a novel hybrid method to reduce speckle noise in ultrasonography. This method applies the total variation denoising algorithm to the output image of a recently reported anisotropic diffusion filter. Performance of the proposed method is illustrated using simulated and clinical images. Experimental results indicate the proposed method outperforms the existing despeckling schemes in terms of both speckle reduction and edge preservation.

## Keywords

Speckle, anisotropic diffusion, total variation denoising.

## 1. INTRODUCTION

Ultrasonography (also called ultrasound imaging) is a widely used diagnostic instrument, which provides clinician with real-time images for diagnosis and therapy. The images acquired from ultrasonography, however, give poor contrast resolution because of the presence of speckle. Speckle is a common undesired phenomenon in any coherent imaging process, which masks details of ultrasound images and affects tasks of image analysis. For a raw ultrasound image, it is often difficult to truthfully interpret the diagnostic information behind the lesion and directly make a distinction between tumor tissue and adjacent tissue, even for a trained radiologist. Therefore, in order to reduce speckle inherent in ultrasound images and help clinician with examination and diagnosis, a novel hybrid despeckling method is proposed in this article. The fundamental idea underpinning this speckle reducing method is adopting the total variation denoising algorithm to the output image of a recently reported speckle reducing anisotropic diffusion filter.

The remainder of this paper is organized as follows. Section 2 reviews the model of speckle. In section 3, anisotropic diffusion filter and total variation denoising algorithm used for reducing speckle are elaborated, and then the proposed anisotropic diffusion followed by total variation denoising (ADFTV) is present. Section 4 is devoted to experimental results on both simulated and clinical images to evaluate the performance of ADFTV. Finally, conclusions are drawn in section 5.

## 2. MODEL OF SPECKLE

An effective despeckling category requires a reasonable and accurate statistical model of speckle. Speckle is a form of locally correlated multiplicative noise. Throughout this paper, the following equation is employed as the generalized model for speckle:

$$I = Jn \quad (1)$$

where  $I$  represents the observed image,  $n$  denotes the noise introduced by the acquisition of ultrasound images, and  $J$  is the noise free image that needs restoring [1].

## 3. METHODS

### 3.1 Anisotropic Diffusion

Anisotropic diffusion was introduced by Perona and Malik (PM), which since then constituted a common and useful tool for image enhancement [2]. Henceforth, lots of achievements on anisotropic diffusion were reported.

#### 3.1.1 Speckle Reducing Anisotropic Diffusion

Based on PM and the adaptive filter, a variation of anisotropic diffusion method called speckle reducing anisotropic diffusion (SRAD), which shows good performance with different levels of speckle, was proposed in [3]. In SRAD, the following partial differential equation (PDE) is considered as the diffusion equivalent:

$$\begin{cases} \partial I(\mathbf{x};t)/\partial t = \text{div}[c(q(\mathbf{x};t)) \cdot \nabla I(\mathbf{x};t)] \\ I(\mathbf{x};0) = I_0(\mathbf{x}), (\partial I(\mathbf{x};t)/\partial \bar{n})|_{\partial \Omega} = 0 \end{cases} \quad (2)$$

where  $I_0(\mathbf{x})$  is the input image at location  $\mathbf{x} = (x, y)$ ,  $\text{div}$  represents the divergence operator,  $\nabla$  denotes the gradient operator,  $\partial \Omega$  is the border of the image domain  $\Omega$ ,  $\bar{n}$  represents the outward normal vector to  $\partial \Omega$ ,  $I(\mathbf{x};t)$  is the output image at time  $t$  and the diffusive coefficient  $c(\square)$  is given by

$$c(q(\mathbf{x};t)) = 1 / \left\{ 1 + \left[ q^2(\mathbf{x};t) - q_0^2(t) \right] / \left[ q_0^2(t) (1 + q_0^2(t)) \right] \right\} \quad (3)$$

Here,  $q(\mathbf{x};t)$  is the instantaneous coefficient of variation, as defined by

$$q(\mathbf{x};t) = \sqrt{\text{Var}(I)/\bar{I}} = \sqrt{(1/|\eta|) / \sum_{p \in \eta} (I_p - \bar{I})^2} \quad (4)$$

where  $\bar{I} = (1/|\eta|) \sum_{p \in \eta} I_p$ ,  $\eta$  and  $|\eta|$  denote the window and the number of neighbors.  $q(\mathbf{x};t)$  turns out to present high values at edges and produce low values in homogeneous regions.  $q_0(t)$  is the speckle scale function to determine the amount of smoothing, which can be estimated by

$$q_0(t) = (C/\sqrt{2}) \text{MAD}(\nabla \ln I(\mathbf{x};t)) \quad (5)$$

where  $\text{MAD}(\nabla \ln I) = \text{median}\{\|\nabla \ln I - \text{median}\{\|\nabla \ln I\|\}\}\}$ ,  $\text{MAD}$  denotes the median absolute deviation and  $\|\cdot\|$  represents the gradient magnitude. The scaling factor  $C$  is set as a constant 1.4826[4,5].

### 3.1.2 New Speckle Reducing Anisotropic Diffusion

Recently, a new speckle reducing anisotropic diffusion model (NSRAD) was investigated in [6]. The NSRAD employs the same PDE as (2), but proposes the following sigmoid function as diffusive coefficient:

$$c(q) = 1 - \frac{1}{1 + \exp\{-k[q^2(\mathbf{x};t) - \beta q_0^2(t)]\}} \quad (6)$$

where  $k$  is adjustable variable to control the attenuation speed of diffusive coefficient,  $\beta$  is called the homogeneous region control coefficient,  $q(\mathbf{x};t)$  and  $q_0(t)$  have the same definition as that of SRAD. In the process of NSRAD filtering,  $q^2(\mathbf{x};t)$  is chosen as a self-correcting parameter to the noise estimation  $q_0^2(t)$  as  $k = K/q_0^2(t)$ , with  $K$  is a suitable constant. It is shown that NSRAD enables the capability of smoothing homogeneous region whilst efficiently avoiding block effect. In this paper, NSRAD model is used as the anisotropic diffusion filter of ADFTV.

### 3.2 Total Variation Denoising

Total variation denoising (TV denoising) algorithm takes into consideration that the signals with excessive and possibly spurious detail have high total variation, so minimizing the total variation is able to remove undesired details. The concept of TV denoising was pioneered by Rudin et al. through the seminal paper [7], in which the following constrained minimization problem is considered:

$$\text{minimize} \int_{\Omega} \sqrt{J_x^2 + J_y^2} dx dy \quad (7)$$

subject to

$$1/|\Omega| \int_{\Omega} (J - I)^2 dx dy = \sigma^2 \quad (8)$$

where  $\Omega$  is a bounded, convex region in two-dimensional space with  $(x, y) \in \Omega$  and  $\sigma$  is the standard deviation of the noise  $n$ .

Then the following Euler-Lagrange equation is arrived:

$$\nabla \cdot (\nabla J / |\nabla J|) + \lambda (I - J) = 0 \quad (9)$$

where  $\lambda$  is a regularized scale parameter and  $|\cdot|$  represents the absolute value. The Euler-Lagrange equation (9) can be solved numerically using a discretized scheme, which is detailed in [7].

### 3.3 ADFTV Method

To perform ADFTV, NSRAD filter is implemented first. As for the instantaneous coefficient of variation  $q(\mathbf{x};t)$  and the speckle scale function  $q_0(t)$ , needed in (6), is calculated using (4) and (5) on a  $3 \times 3$  neighborhood. When undergo a NSRAD filtering, the resulting images are good candidates for TV denoising. Thus, in the next step TV denoising algorithm is applied to the output image of NSRAD filtering.

## 4. EXPERIMENTS AND DISCUSSIONS

The performance of ADFTV is evaluated using both simulated and clinical images. The simulated studies give quantitative performance assessment, and in clinical studies demonstrate the usefulness and practicability of ADFTV in real applications.

### 4.1 Results on Simulated Images

The proposed ADFTV method and some various methods are tested on the Lena image corrupted by a uniformly distributed multiplicative noise, with variance is 0.0833 (see Figure 1b). Figure 1a is the original clean image, Figure 1c-f show the corresponding processed results of PM, SRAD, NSRAD, and ADFTV. Time step is chosen to be  $\Delta t = 0.1$ , and 100 iterations are implemented. Besides,  $K = 2$  and  $\beta = 1$  are empirically set for the best visual quality.



**Fig 1: Results on Lena image with  $\Delta t = 0.1$ : (a) original clean image, (b) noise image, (c-f) processed results of PM, SRAD, NSRAD and ADFTV, respectively.**

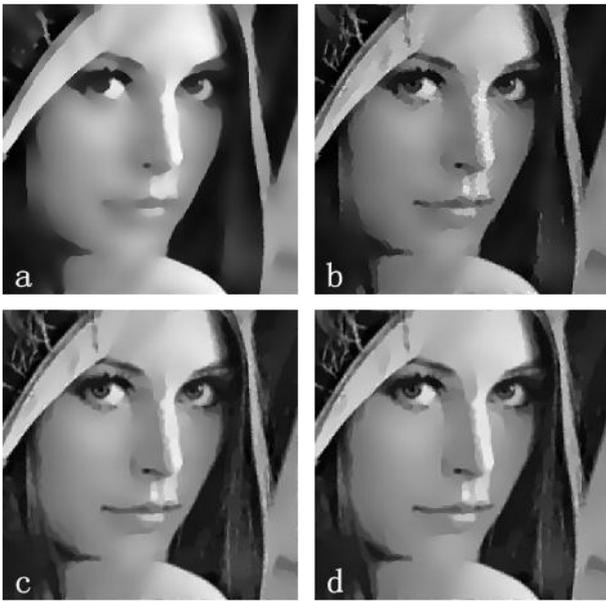


Fig 2: Results on Lena image with different  $\Delta t$  : (a~b) processed results of SRAD and NSRAD with  $\Delta t = 0.5$  , (d~f) processed results of ADFTV with  $\Delta t = 0.5, 1$  , respectively.

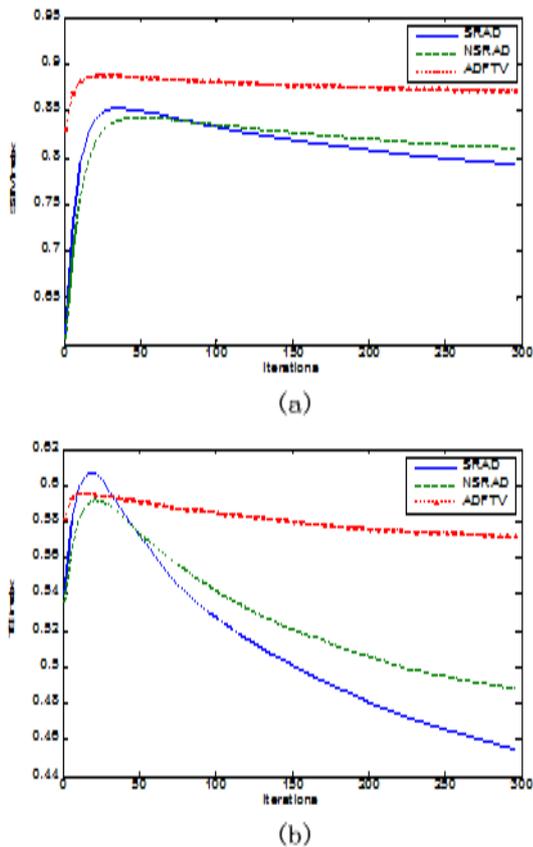


Fig 3: SSIM and TEI comparison of different methods on Lena image: (a) plots of SSIM index versus iteration, (b) plots of TEI index versus iteration.

As seen from Figure 1, PM and SRAD blur some of the edges. NSRAD shows some residual noise. But, ADFTV gives the most effective despeckling performance and presents the best visualized appearance related to the original clean image.

This experiment is repeated for greater values of  $\Delta t$  in Figure 2. One can see that SRAD is unstable, even for  $\Delta t = 0.5$  , a result that seems not to match the authors' statement in [3]. NSRAD still shows some residual noise. But ADFTV gives better visual quality than other methods without blurring edges, even for  $\Delta t = 1$  .

Additionally, some quality assessment indices are employed to confirm the performance of ADFTV, including structural similarity index (SSIM) [8] and transferred edge information (TEI) [9]. The plots of SSIM and TEI index versus number of iterations are shown in Figure 3 with  $\Delta t = 0.1$  . In Figure 3a, the ADFTV gives better SSIM index than other methods. This indicates the proposed ADFTV method preserves more structure features of the original clean image. As for the TEI index, which is shown in Figure 3b, NSRAD is better than SRAD, while the ADFTV has the largest TEI value. This improvement on TEI index corresponding to better performance on edge information preservation of the proposed ADFTV method even becomes more prominent with the increasing of iterations.

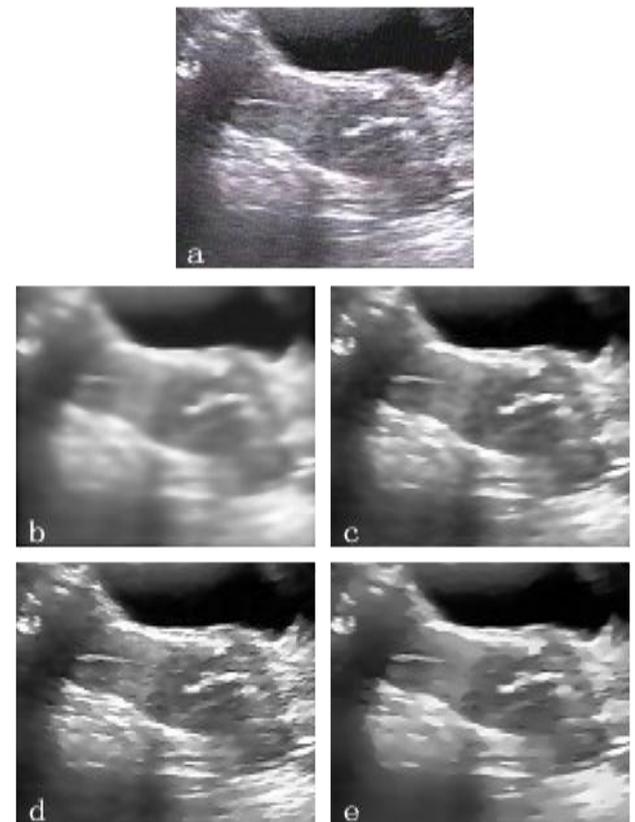


Fig 4: Results on a clinical ultrasound image regarding to cervix cancer: (a) raw image, (b~e) processed results of PM, SRAD, NSRAD and ADFTV, respectively.



## 4.2 Results on Clinical Images

The proposed ADFTV method is further tested on real clinical ultrasound images regarding to cervix cancer, which are acquired from the ACUSON X300 ultrasound system manufactured by SIEMENS. Figure 4 shows a representative and typical raw ultrasound image of cervical cancer and the corresponding processed results of PM, SRAD, NSRAD, and ADFTV. The same estimation procedure in the simulated studies is employed for the experiments on clinical ultrasound images. Time step is chosen to be  $\Delta t = 0.1$ , 100 iterations are run, but  $K = 400, \beta = 0.8$  are performed. Before processing these images, many sections of useless information have been cut off. As shown in Figure 4, it is observed that the ADFTV method reveals better diffusion performance and preserves sharper boundaries from the visual effect apparently than the other methods.

## 5. CONCLUSIONS

In this paper, a novel hybrid method called ADFTV is proposed for despeckling in ultrasonography. This proposed method applies TV denoising algorithm to the output image of NSRAD filter. Experimental results on both simulated and clinical images are present to evaluate the performance of ADFTV, which is superior to the conventional anisotropic diffusion techniques.

## 6. REFERENCES

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