



Nonlinear Diffusion based on Bayesian Estimator in Laplacian Pyramid Domain for Ultrasonic Speckle Reduction

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ABSTRACT

Laplacian pyramid based nonlinear diffusion (LPND) method is proposed for speckle noise reduction in medical ultrasound imaging. In this method, speckle is removed by nonlinear diffusion filtering of bandpass ultrasound images in Laplacian pyramid domain. For nonlinear diffusion in each pyramid layer, a Bayesian threshold is automatically determined by a variation of robust median estimator. The proposed LPND method reduces speckle noise significantly while preserving sharp image details. The superiority of the proposed LPND over the conventional LPND based on Gaussian filtering and LPND method based on Butterworth filtering based speckle reduction technique is demonstrated by applying them on synthetic and real ultrasound B-mode images.

Keywords

Laplacian pyramid, multiscale analysis, nonlinear diffusion, speckle reduction, Bayesian estimator, ultrasound imaging.

1. INTRODUCTION

Ultrasound imaging has become a popular modality because it is safe, noninvasive, portable, relatively inexpensive, and provides a real-time image formation. The diagnostic usefulness of ultrasound imaging is at times limited due to its low image quality. One of the main reasons for this low image quality is the presence of signal-dependent noise known as speckle. Speckle is a granular pattern formed due to constructive and destructive coherent interferences of backscattered echoes from the scatterers that are typically much smaller than the spatial resolution (i.e., wavelength of an ultrasound wave) of medical ultrasound systems [1]. Speckle degrades the image quality, and hence reduces the ability of a human observer to discriminate the fine details in diagnostic examination, and also degrades the speed and accuracy of ultrasound image processing tasks. When applying a speckle reduction technique as an aid for visual diagnosis, we need to keep in mind that certain speckle contains diagnostic information and should be retained. Also, some important details may be smeared or lost when performing speckle reduction. In some cases, clinicians prefer an original image to a despeckled image, because the original image contains more diagnostic information. From this point of view, the despeckled image should be considered a complement to the original image, and not a replacement. When speckle reduction is applied as a preprocessing step for segmentation or registration, any speckle can be considered noise without differentiation. In these applications, speckle prevents image segmentation and registration techniques from generating optimal results. Speckle reduction makes an ultrasound image

cleaner with clearer boundaries, and thus significantly improves the speed and accuracy of automatic or semiautomatic image segmentation and registration techniques [1]. Various speckle reduction techniques have been proposed[4]-[12], namely, compounding techniques, which try to reduce speckle by generating multiple uncorrelated images, cause the loss of small details because of blurring. Wavelet multiscale analysis has the property of space and scale localization, and has found successful applications in a variety of signal processing problems. Another multiscale representation is Laplacian pyramid [5]. In the Laplacian pyramid decomposition scheme, a signal is successively decomposed into a decimated lowpass signal and a bandpass signal. The bandpass signal is the difference between the signal on a finer scale and the interpolated signal from a coarser scale [1]. In this paper, we present a new speckle reduction method based on Laplacian pyramid. Speckle reduction is achieved by nonlinear diffusion of the individual bandpass levels of the Laplacian pyramid representation of an image. In section two, we will discuss briefly Laplacian pyramid and nonlinear diffusion theory. Section three describes our proposed method. Experimental results and discussion are given in section four and the conclusion is in section five.

2. THEROTICAL BACKGROUND

2.1 Laplacian Pyramid

Laplacian Pyramid is the band pass filter -each level represents spatial frequencies (largely) unrepresented at other levels. It preserves difference between upsampled Gaussian pyramid level and Gaussian pyramid level. Over-complete decomposition based on difference-of-lowpass filters; the image is recursively decomposed into low-pass and highpass bands. Laplacian pyramid is orientation independent. The first step in Laplacian pyramid coding is to low-pass filter the original image G_0 to obtain image G_1 . So G_1 is a "reduced" version of G_0 [5]. In a similar way we form G_2 as a reduced version of G_1 , and so on. Filtering is performed by a convolution weighting functions. The sequence of G_0, G_1, \dots, G_n is called Gaussian pyramid. The level-to-level averaging process is performed by the function REDUCE. The REDUCE operator performs a two-dimensional lowpass filtering followed by a sub-sampling by factor of two in both directions[5]. For an image I,

$$G_i = \text{REDUCE}[G_{i-1}] \dots \dots \dots (1)$$

We now define a function EXPAND as the reverse of REDUCE[1]. Its effect is to expand an $(M + 1)$ -by- $(N + 1)$ array into a $(2M + 1)$ -by- $(2N + 1)$ array by interpolating new



node values between the given values. Let $G_{i, n}$ be the result of expanding $G_{i, n-1}$ times. Then

$$G_{i, n} = \text{EXPAND} (G_{i, n-1}) \dots\dots\dots(2)$$

The Laplacian pyramid is a sequence of error images L_0, L_1, \dots, L_N . Each is the difference between two levels of the Gaussian pyramid. Thus, for $0 < 1 < N$. The EXPAND operator enlarges an image to twice the size in both directions by up-sampling (insertion of zeros) and a low-pass filter followed by a multiplication by a factor of four[1]. Then the Laplacian pyramid defined as

$$L_i = G_i - \text{EXPAND}[G_{i-1}] \dots\dots\dots(3)$$

The Gaussian pyramid consists of a set of lowpass filtered copies of the original image at different sizes, whereas the Laplacian pyramid decomposes the original image into a set of bandpass images and a final lowpass image. Reconstruction of an image from its Laplacian pyramid can be achieved by simply reversing the decomposition steps [5].

2.2 Nonlinear Diffusion

Diffusion filtering removes noise from an image by modifying the image via a partial differential equation (PDE)[1]. The linear diffusion equation is given by

$$\frac{\partial I}{\partial t} = \text{div}[c \cdot \nabla I], I(t=0)=I_0 \dots\dots\dots(4)$$

where div is the divergence operator, ∇I is the image gradient, c is the diffusion function, and I_0 is the original image. Perona and Malik [7] proposed the nonlinear diffusion as described by the following equation:

$$\frac{\partial I}{\partial t} = \text{div}[c(\|\nabla I\|) \cdot \nabla I], I(t=0)=I_0 \dots\dots\dots(5)$$

where div is the divergence operator, $\|\nabla I\|$ is the gradient magnitude of an image I , $c(\|\nabla I\|)$ is the diffusion coefficient or diffusivity function, and I_0 is the original image. If the function $c(\|\nabla I\|)$ is constant for all image locations, the diffusion process becomes linear. For nonlinear diffusion, the diffusivity function $c(\|\nabla I\|)$ is a monotonically decreasing function of the gradient magnitude. Perona and Malik [7] suggested two diffusivity functions

$$c(\|\nabla I\|) = \frac{1}{1 + (\|\nabla I\|/k)^2} \dots\dots\dots(6)$$

$$c(\|\nabla I\|) = \exp[-(\|\nabla I\|/k)^2] \dots\dots\dots(7)$$

where k is gradient threshold. It plays an important role in determining the parts of the image that will be blurred or enhanced in the diffusion process.

The basic idea of LPND is replace the estimated gradient ∇I by a Gaussian smoothed version[1].

$$\frac{\partial I}{\partial t} = \text{div}[c(\|\nabla(G(\partial) * I)\|) \cdot \nabla I] \dots\dots\dots(8)$$

Where ∂ is the standard deviation of a Gaussian filter G .

3. THE PROPOSED METHOD

Noise reduction method based on Laplacian pyramid is similar to those wavelet based denoising methods. They consist of three procedures: transformation of the image into its Laplacian pyramid structure, manipulation on pyramidal coefficients, and then reconstruction of the pyramid[1]. After an image is decomposed into its pyramid structure of decreasing frequencies, Speckle noise and useful signal components of the image exist in different layers because of their different frequency characteristics. Speckle noise has high frequency so that it mainly exists in low pyramid layers (i.e., fine scales). On the other hand, in the highest pyramid layer (i.e. the coarsest scale), speckle noise is negligible. Thus, performing spatial adaptive filtering in each bandpass layer can effectively suppress speckle without degrading slowly varying signal too much. With a certain criterion, the pyramidal coefficients are divided into two groups. The first group contains important, regular coefficients; while the second group consists of coefficients that are considered as irregular and caused mainly by noise. Noise can be removed by suppressing those irregular pyramidal coefficients. And some image features can be enhanced by selectively amplifying those important transform coefficients that represent features of interest. In this way, noise reduction and image enhancement can be achieved simultaneously. The first proposed nonlinear diffusion process has the property of sharpening edges while reducing noise in smooth areas at the same time. The basic idea of LPND is replace the estimated gradient ∇I by Butterworth lowpass filtering(smoothed) version.

$$\frac{\partial I}{\partial t} = \text{div}[c(\|\nabla(B(D_0) * I)\|) \cdot \nabla I] \dots\dots\dots(9)$$

Where D_0 is the cut-off-frequency of a Butterworth lowpass filter B .

The another proposed method consists of the following three steps. First, we decompose an image into its Laplacian pyramid structure; then apply a regularized nonlinear diffusion process on each pyramid levels except the highest one which is a lowpass approximation of the input image; and finally reconstruct the pyramid to get the denoised image. The second step is the most crucial, and it distinguishes our method from other Laplacian pyramid based noise reduction methods. The nonlinear diffusion process in the second step is a Bayesian regularized process. Using a diffusion coefficient like described nonlinear diffusion may create problems such as unsolved existence and high sensitivity to noise. Some regularization methods have been proposed. The basic idea is to replace the estimated Bayesian Smoothed version. Thus the equation (5) becomes



$$\frac{\partial I}{\partial t} = \text{div}[c(|\nabla(\text{Bayes}(\partial) * I)|) \cdot \nabla I] \dots\dots\dots (10)$$

where ∂ is the noise standard deviation of Bayesian estimator filter Bayes. To get Bayesian estimator first we do wavelet decomposition using Haar transformation. Then calculate the ∂ from the well-known robust median estimator,

$$\partial = \frac{\text{median}(|\text{HH1 and HH2}|)}{0.6745} \dots\dots\dots (11)$$

Where, HH₁ and HH₂ are the set of wavelet coefficients of the first and second level diagonal detail subbands in the wavelet domain, and the constant 0.6745 is unit variance. And then perform soft-thresholding and reconstruct the image from the Bayes-thresholded wavelet coefficients. In conventional nonlinear diffusion methods, the gradient threshold is mainly determined by experience depending mainly on the noise level. When the noise level is unknown, a test series with different parameters must be generated and compared. Once selected, the parameter can be kept fixed for images acquired under similar conditions. However, such a procedure is very time consuming and unsuitable for clinical applications. Currently, we estimate the Bayesian threshold using the robust median estimator. Our experiments show that such an estimate is close to the optimal value in most cases.

4. EXPERIMENTAL RESULTS AND DISCUSSION

In each experiment, we compared the results of LPND against those of a LPND method using Gaussian lowpass filtering, its diffusion coefficient is computed using follows equation(8) and LPND method using Butterworth lowpass filtering, its diffusion coefficient is computed using follows equation (9).The gradient threshold is determined by experience in all method. The filter window is 5 by 5 at all pyramidal levels. We used four decomposition levels. Using more levels did not guarantee improved performance.

The synthetic image is represented by

$$I(x, y) = S(x, y) \cdot N(x, y) \dots\dots\dots (12)$$

Where S(x, y) is the reference noise free image and N(x, y) is a random process representing the variations in speckle amplitude.

In order to quantify the achieved performance improvement, two measures as used in [7] were computed based on the reference noise-free and the denoised image. One measure is the mean squared error (MSE), which is used to evaluate the noise reduction ability.

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^N (S_i - s_i)^2 \dots\dots\dots (13)$$

where S is the reference image which is considered to noise free; s is denoised image, and N represents the total number of pixels in the image.

We chose a noisy ultrasound image, applied the methods and visually evaluated the denoised image. The results are given in Fig.1; we magnified a part of the image for better visualization. The first proposed method that based on Butterworth lowpass filtering removed more speckle than LPND method based on Gaussian and second proposed method that based on LPND using Bayesian estimator removed more speckle than LPND method based on Gaussian and Butterworth lowpass filtering .

5. CONCLUSION

A new nonlinear diffusion method in Laplacian pyramid domain for ultrasonic speckle reduction has been investigated. It consists of the following three steps. First, the input image is decomposed into its Laplacian pyramid domain. Then, a regularized nonlinear diffusion process is performed in each pyramid layer except the highest one to remove the speckle. Finally, the diffused Laplacian pyramid is reconstructed to get the despeckled image. An automatic estimation of the Bayesian threshold by robust median estimator is also integrated in the LPND to make the algorithm practical. Experimental results on synthetic and real ultrasound B-mode images show that the proposed method can reduce speckle noise significantly while preserving sharp image details. It can be further improved by a better choice of the Bayesian threshold rule.

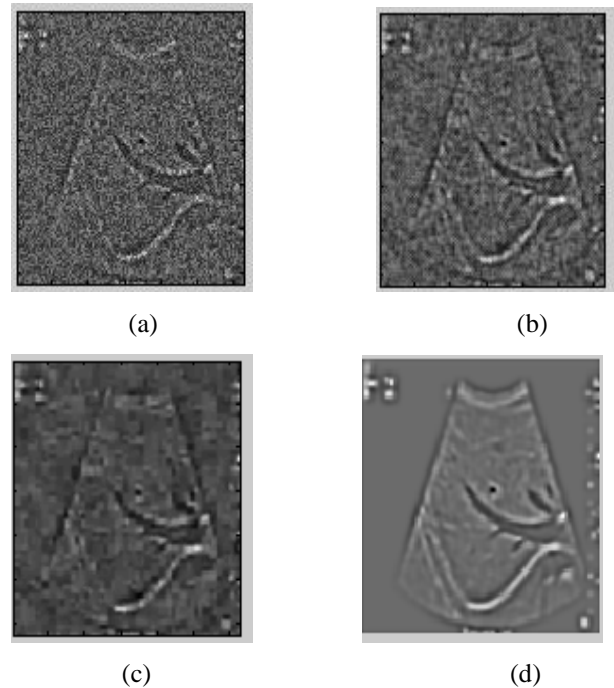


Fig.1.(a) Original noisy image (b) LPND using Gaussian filtering (c) LPND using Butterworth filtering (d) LPND using Bayesian estimator.



TABLE I: SNR, PSNR, and MSE values for the three methods on a synthetic image

	SNR	PSNR	MSE
Without filtering	5.6427	25.9148	166.5697
LPND using Gaussian filtering	7.4246	24.0729	35.2928
LPND using Butterworth filtering	7.4779	24.0713	35.3003
LPND using Bayesian estimator	12.7491	75.2978	0.0019

6. REFERENCE

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