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Order Reducing of Linear System using Clustering Method Factor Division Algorithm

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ABSTRACT

A mixed method is proposed for finding stable reduced order models of single-input- single-output large-scale systems using Factor division algorithm and the clustering technique. The denominator polynomial of the reduced order model with respect to original model is determined by forming the clusters of the poles of the original system, and the coefficients of numerator polynomial with respect to original model are obtained by using the Factor division algorithm. The mixed methods are simple and guarantee the stability of the reduced model if the original system is stable. The methodology of the proposed method is illustrated with the help of examples from literature.

Keywords

Clustering technique, Order reduction, Factor division algorithm, Transfer function, Stability, Integral square error (ISE).

1. INTRODUCTION

The modeling of complex dynamics is one of the most important subjects in engineering. Moreover a model is often too complicated to be used in real problems, so approximation procedures based on physical considerations or using mathematical approaches must be used to achieve simple models than original ones. The order reduction technique in both frequency and time domain has been proposed by several authors[1-8].Further, several mixed method have been suggested for order reduction by authors[9-13].In spite of having these order reduction techniques, it is seen that none always gives the best result. The proposed method is a mixed method for order reduction of large-scale single-input-single-output (SISO) system which combines the Clustering technique and Factor division algorithm. The denominator of the reduced order with respect to the original model is synthesized by clustering technique [14], in which clusters are formed by combing the poles of the which clusters are formed by using Factor division algorithm[15].The Factor division algorithm can be used to determining reduced numerator. The Factor division algorithm has been successfully used to find reduced order approximants of high-order systems. It avoids finding time moments and solving the Pade equation, whilst the reduced models still retain the initial time moments of the full systems. It is shown how the concept of factor division emerged from the method of shamash and consequently how the algorithm may be used in other methods to ease computation.

2. STATEMENT OF PROBLEM

Consider n[th] an order linear dynamic single-input single-output system, described by the transfer function

$$G(sv) = \frac{N(s)}{D(s)} = \frac{a_0 + a_1(s) + a_2 s^2 + \cdots \ldots a_{n-1} s^{n-1}}{b_0 + b_1(s) + b_2 s^2 + \cdots \ldots b_{n-1} s^{n-1} + b_n s^n} \qquad 1$$

where $a_i; 0 \le i \le n-1$ and $b_i; 0 \le i \le n$ are scalar constants.

The corresponding k^{th} $k < n$ order reduced model is synthesized as

$$G_r(s) = \frac{d_0 + d_1(s) + d_2 s^2 + \cdots d_{n-1} s^{n-1}}{e_0 + e_1(s) + e_2 s^2 + \cdots e_{n-1} s^{n-1} + e_n s^n} \qquad 2$$

where $d_i; 0 \le i \le n-1$ and $e_i; 0 \le i \le n$ are scalar constants.

The objective of this paper is to realize k^{th} order reduced model in the form of (2) from (1), such that it retains the important features of the original system and approximates its step response as close as possible.

3. DESCRIPTION OF THE METHOD

This method consists of the following two steps (1and 2):

Steps 1 *Determination of the denominator of* k^{th} *order reduced model, using the clustering technique* [14].

The criterion for grouping the poles in one particular cluster is based on relative distance between the poles and the desired order in the process of reduced order modeling. Since each cluster may be finally replaced by a single (pair of) real (complex) pole, the following rules are used for clustering the poles to get the denominator polynomial for reduced order models.

i. Separate clusters should be made for real poles and complex poles.
ii. Poles on the jw-axis have to be retained in the reduced order model.

The cluster centre can be formed using a simple method known as 'inverse distance measure', which is explained as follows:

Let, *r* real poles in one cluster be ($p_1, p_2, p_3, \ldots \ldots . p_n$) then the Inverse Distance Measure (IDM) criterion identifies the cluster centre as

$$p_c = \{(\textstyle\sum_{i=1}^{r}(1/p_i)) \div r\}^{-1} \qquad 3$$

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where p_c is cluster centre from *r* real poles of the original system.

Let, m pair of complex conjugate poles in a cluster be $[(\propto_1 \pm j\beta_1)(\propto_2 \pm j\beta_2) \dots\dots\dots\dots\dots(\propto_m \pm j\beta_m)]$ then the IDM criterion identifies the complex cluster centre in the form $(A_c \pm jB_c)$.

$$A_c = \{(\sum_{i=1}^{m}(1/\propto_i)) \div m\}^{-1}$$

and

$$B_c = \{(\sum_{i=1}^{m}(1/\beta_i)) \div m\}^{-1} \quad 4$$

For getting the denominator polynomial for k^{th} order reduced model, one of the following cases may occur.

Case 1: If all cluster centers are real, then denominator polynomial for k^{th} order reduced model can be obtained as

$$D_k(s) = (s - p_{c1})(s - p_{c2}) \dots\dots\dots\dots(s - p_{ck}) \quad 5$$

where p_{c1}, p_{c2},………. p_{ck} are 1st ,2nd,……. k^{th} cluster centre.

Case 2: If (k-2) cluster centers are real and one pair of cluster centre is complex conjugate, then $D_k(s)$ can be obtained as

$$D_k(s) = (s - p_{c1})(s - p_{c2}) \dots (s - p_{c(k-2)})(s - \dot{p}_{c1})(s - \hat{p}_{c2}) \quad 6$$

where $\dot{p}_{c1}$ and $\hat{p}_{c2}$ are complex conjugate cluster centers or

$\dot{p}_{c1} = (A_c + jB_c)$ and $\hat{p}_{c2} = (A_c - jB_c)$

Case 3: If all the cluster centers are complex conjugate, then reduced denominator polynomial can be taken as

$$D_k(s) = (s - \dot{p}_{c1})(s - \hat{p}_{c1})(s - \dot{p}_{c2})(s - \hat{p}_{c2}) \dots\dots\dots (s - \dot{p}_{ck/2})(s - \hat{p}_{ck/2}) \quad 7$$

***Steps 2** Determination of the numerator of k^{th} order reduced model using Factor Division algorithm* [15].

After obtaining the reduced denominator, the numerator of the reduced model is determined,

$$\tilde{N}(s) = = \frac{N(s)}{D(s)} * D_k(s) = \frac{N(s)}{D(s)/D_k(s)} \quad 8$$

where $D_k(s)$ is reduced order denominator.

Therefore, the numerator $\tilde{N}(s)$ of the reduced order model $G_r(s)$in Eq. (2) will be the series expansion of

$$\frac{N(s)}{\frac{D(s)}{D_k(s)}} = \frac{\sum_{i=0}^{n-r} a_i s^i}{\sum_{i=0}^{n-r} e_i s^i}$$

about s =0 up to the term of order s^{r-1}.

This is easily obtained by modified of moment generating[15],which uses the familiar Routh recurrence formula to generate the third, fifth, seventh, etc., rows as,

$\propto_0 = \frac{a_0}{e_0}$ $\quad a_0 \quad a_1 \quad a_2 \dots \quad a_{r-1}$

$e_0 \quad e_1 \quad e_2 \quad \dots\dots e_{r-1}$

$\propto_1 = \frac{q_0}{e_0}$ $\quad q_0 \quad q_1 \quad \dots\dots\dots\dots q_{r-2}$

$e_0 \quad e_1 \quad \dots\dots\dots\dots\dots e_{r-2}$

$\propto_2 = \frac{r_0}{e_0}$ $\quad r_0 \quad r_1 \quad \dots\dots\dots\dots\dots r_{r-3}$

$e_0 \quad e_1 \dots\dots\dots \quad \dots\dots e_{r-3}$

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$\propto_{r-2} = \frac{u_0}{e_0}$ $\quad u_0 \quad u_1$

$e_0 \quad e_1$

$\propto_{r-1} = \frac{v_0}{e_0}$ $\quad v_0$

e_0

where

$q_i = b_{i+1} - \propto_0 e_{i+1}, i = 0; 1; \dots; r - 3.$

$r_i = q_{i+1} - \propto_1 e_{i+1}, i = 0; 1; \dots; r - 3.$

………………….

………………….

$v_0 = u_1 - \propto_{r-2} e_1$

Therefore, the numerator $\tilde{N}(s)$ of Eq. (2) is given by

$$\tilde{N}(s) = \sum_{i=0}^{n-r} \propto_i s^i$$

4. METHOD FOR COMPARISON

In order to check the accuracy of the proposed method the relative integral square error ISE index in between the transient parts of the reduced models and the original system is calculated using Matlab/Simulink. The relative integral square error RISE is defined as

$$ISE = \int_0^{\infty} [y(t) - y_r(t)]^2 \, dt$$

5. NUMERICAL EXAMPLES

Example-1: Consider sixth order system

$$G_6(s) = \frac{N(s)}{D(s)}$$

$N(s) = s^5 + 1014s^4 + 14069s^3 + 69140s^2 + 140100s + 100000$

$D(s) = s^6 + 222s^5 + 14541s^4 + 248420s^3 + 1454100s^2 + 2220000s + 1000000$

Using step-1, two cluster centers from the real polees-1,-1,-10,-10,-100,-100 can be formed equation (3),

$P_{c1} = 0.9523$

$P_{c2} = 16.66$

Therefore, denominator $D_k(s)$ can be synthesized using equation (5) and is given by

$D_2(s) = (s + 0.9523)(s + 16.66)$

$D_2(s) = 15.86 + 17.61s + s^2$

Using Factor division algorithm, the numerator is determined as

$\tilde{N}(s) = 1.5860 + 0.4620s$

Therefore, the second order reduced model is taken as

$$G_2(s) = \frac{1.5860 + 0.4620s}{15.86 + 17.61s + s^2}$$

ISE=1.7076$\times 10^{-6}$

The comparison of step responses of the 2nd order model and the sixth order system is shown in figure1.

Example-2: Consider fourth order stable system

$$G_4(s) = \frac{2400 + 1800s + 496s^2 + 28s^3}{240 + 360s + 204s^2 + 36s^3 + 2s^4}$$

Poles: -7.8033$\pm$j1.3576, -1.1967$\pm$0.6934

From these poles, one complex cluster centre is calculated, which is given as,

$A_1 \pm jB_1 = -2.07516 \pm j0.9179$

Then reduced order denominator using equation (7) as

$D_2(s) = 5.14 + 4.150s + s^2$

Using Factor division algorithm, the numerator is determined as

$\tilde{N}(s) = 51.48 + 2.80s$

Therefore, the second order reduced model is taken as

$$G_2(s) = \frac{51.48 + 2.80s}{5.14 + 4.150s + s^2}$$

ISE=0.005108

The comparison of step responses of the 2nd order model and the sixth order system is shown in figure2

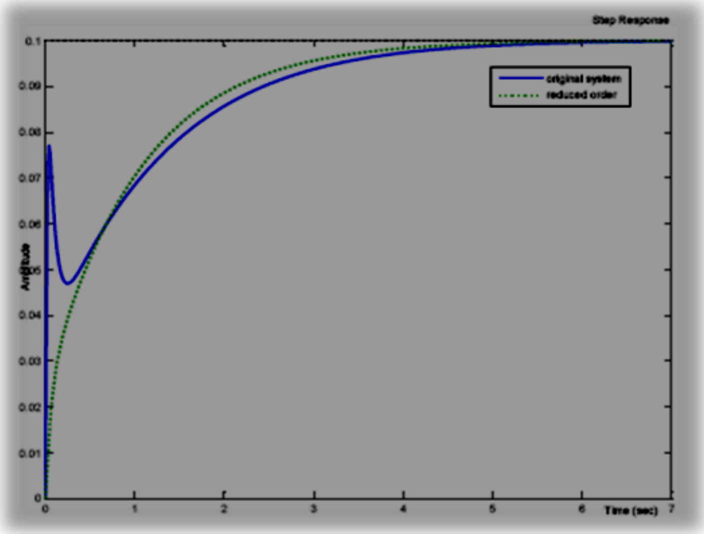

Figure 1 Step response of original system and ROM of example 1

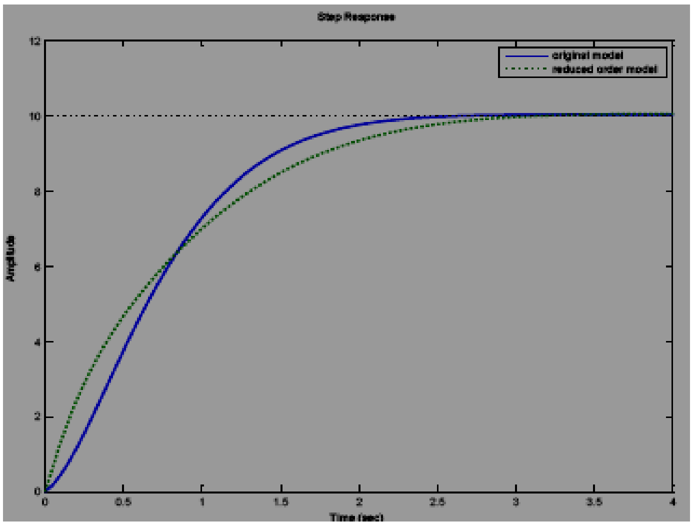

Figure 2 Step response of original system and ROM of example 2

6. CONCLUSIONS

A mixed method for order reduction of SISO stable system has been presented, based on clustering technique and Factor division algorithm. In this method, the denominator of the reduced model is synthesized by using clustering technique in which poles are grouped into several clusters. These clusters are then replaced by the respective cluster-centers. The coefficients of the numerator polynomial are obtained by Factor division algorithm. The proposed method is illustrated with the help of two examples and it has been observed that this method gives better than many other existing order reduction techniques available in literature. The method is simple, efficient and takes little computational time, but, in some cases, the reduced model has a tendency to become non-minimum phase. This mixed method preserves model stability and avoids steady-state error between the step responses of the original and reduced models.

This method has been extended for multivariable linear system and is reported elsewhere.

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