



Intuitionistic Fuzzy Completely Regular Weakly Generalized Continuous Mappings

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ABSTRACT

In this paper we introduce intuitionistic fuzzy completely regular weakly generalized continuous mappings and some of their properties are studied.

Key words and phrases

Intuitionistic fuzzy topology, intuitionistic fuzzy regular weakly generalized closed set, intuitionistic fuzzy regular weakly generalized open set, intuitionistic fuzzy regular weakly generalized continuous mappings and intuitionistic fuzzy completely regular weakly generalized continuous mappings.

AMS SUBJECT CLASSIFICATION
MSC 2000: 54A40, 54D20.

1. INTRODUCTION

After the introduction of fuzzy sets by Zadeh¹² in 1965 and fuzzy topology by Chang² in 1967, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov¹ as a generalization of fuzzy sets. In 1997 Coker³ introduced the concept of intuitionistic fuzzy topological spaces. In this present paper we introduce and study the concepts of intuitionistic fuzzy completely regular weakly generalized continuous mappings in intuitionistic fuzzy topological space.

2. PRELIMINARIES

Definition¹ 2.1: An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ where the functions $\mu_A(x): X \rightarrow [0, 1]$ and $\nu_A(x): X \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A , respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by $\text{IFS}(X)$, the set of all intuitionistic fuzzy sets in X .

Definition¹ 2.2: Let A and B be IFS's of the form

$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$. Then

- (a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
- (b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
- (c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$,
- (d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$,

$$(e) A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}.$$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$.

The intuitionistic fuzzy sets $0_- = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1_- = \{ \langle x, 1, 0 \rangle / x \in X \}$ are respectively the empty set and the whole set of X .

Definition³ 2.3: An intuitionistic fuzzy topology (IFT in short) on a non empty X is a family τ of IFS in X satisfying the following axioms:

- (a) $0_-, 1_- \in \tau$
- (b) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$
- (c) $\cup G_i \in \tau$ for any arbitrary family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X .

The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X .

Definition³ 2.4: Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X . Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by $\text{int}(A) = \cup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \}$, $\text{cl}(A) = \cap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}$.

Note that for any IFS A in (X, τ) , we have $\text{cl}(A^c) = [\text{int}(A)]^c$ and $\text{int}(A^c) = [\text{cl}(A)]^c$.

Definition⁶ 2.5: An IFS $A = \{ \langle x, \mu_A, \nu_A \rangle \}$ in an IFTS (X, τ) is said to be an

- (i) intuitionistic fuzzy semi open set (IFSOS in short) if $A \subseteq \text{cl}(\text{int}(A))$,
- (ii) intuitionistic fuzzy α -open set (IF α OS in short) if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$,
- (iii) intuitionistic fuzzy regular open set (IFROS in short) if $A = \text{int}(\text{cl}(A))$.

The family of all IFOS (respectively IFSOS, IF α OS, IFROS) of an IFTS (X, τ) is denoted by $\text{IFO}(X)$ (respectively $\text{IFSOS}(X)$, $\text{IF}\alpha\text{O}(X)$, $\text{IFRO}(X)$).



Definition⁶ 2.6: An IFS $A = \langle X, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

- (i) intuitionistic fuzzy semi closed set (IFSCS in short) if $\text{int}(\text{cl}(A)) \subseteq A$,
- (ii) intuitionistic fuzzy α -closed set (IF α CS in short) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$,
- (iii) intuitionistic fuzzy regular closed set (IFRCS in short) if $A = \text{cl}(\text{int}(A))$.

The family of all IFCS (respectively IFSCS, IF α CS, IFRCS) of an IFTS (X, τ) is denoted by IFC(X) (respectively IFSC(X), IF α C(X), IFR(X)).

Definition⁷ 2.7: An IFS $A = \langle X, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an intuitionistic fuzzy regular weakly generalized closed set (IFRWGCS in short) if $\text{cl}(\text{int}(A)) \subseteq U$, whenever $A \subseteq U$ and U is an IFROS in X .

Result⁷ 2.8: Every IFCS, IFRCS, IFGCS, IFPCS, IF α CS, IF α GCS is an IFRWGCS but the converses may not be true in general.

Definition⁹ 2.9: An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ) .

Definition⁹ 2.10: An IFS A is said to be an intuitionistic fuzzy generalized semi open set (IFGSOS in short) in X if the complement A^c is an IFGSCS in X .

The family of all IFGSCSs (IFGSOSs) of an IFTS (X, τ) is denoted by IFGSC(X) (IFGSO(X)).

Definition⁹ 2.11: Let A be an IFS in an IFTS (X, τ) . Then

$$\text{sint}(A) = \cup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \},$$

$$\text{scl}(A) = \cap \{ K / K \text{ is an IFSCS in } X \text{ and } A \subseteq K \}.$$

Note that for any IFS A in (X, τ) , we have $\text{scl}(A^c) = (\text{sint}(A))^c$ and $\text{sint}(A^c) = (\text{scl}(A))^c$.

Result¹⁰ 2.12: Let A be an IFS in (X, τ) , then

- (i) $\alpha \text{cl}(A) = A \cup \text{cl}(\text{int}(\text{cl}(A)))$,
- (ii) $\alpha \text{int}(A) = A \cap \text{int}(\text{cl}(\text{int}(A)))$.

Definition¹¹ 2.13: An IFS A in an IFTS (X, τ) is an

- (i) intuitionistic fuzzy generalized closed set (IFGCS in short) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X .
- (ii) intuitionistic fuzzy regular generalized closed set (IFRGCS in short) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in X .

Definition⁶ 2.14: An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy alpha generalized closed set (IF α GCS in short) if $\alpha \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ) .

Definition⁶ 2.15: An IFS A is said to be an intuitionistic fuzzy alpha generalized open set (IF α GOS in short) in X if the complement A^c is an IF α GCS in X .

The family of all IF α GCSs (IF α GOSs) of an IFTS (X, τ) is denoted by IF α GC(X) (IFGSO(X)).

Definition⁶ 2.16: Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be intuitionistic fuzzy continuous (IF continuous in short) if $f^{-1}(B) \in \text{IFO}(X)$ for every $B \in \sigma$.

Definition⁶ 2.17: Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an

- (i) intuitionistic fuzzy semi continuous (IFS continuous in short) if $f^{-1}(B) \in \text{IFSO}(X)$ for every $B \in \sigma$.
- (ii) intuitionistic fuzzy α continuous (IF α continuous in short) if $f^{-1}(B) \in \text{IF}\alpha\text{O}(X)$ for every $B \in \sigma$.
- (iii) intuitionistic fuzzy pre continuous (IFP continuous in short) if $f^{-1}(B) \in \text{IFPO}(X)$ for every $B \in \sigma$.

Definition⁵ 2.18: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy γ continuous (IF γ continuous in short) if $f^{-1}(B)$ is an IF γ OS in (X, τ) for every $B \in \sigma$.

Definition² 2.19: Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy generalized continuous (IFG continuous in short) if $f^{-1}(B) \in \text{IFGCS}(X)$ for every IFCS B in Y .

Result² 2.20: Every IF continuous mapping is an IFG continuous mapping.

Definition⁹ 2.21: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy generalized semi continuous (IFGS continuous in short) if $f^{-1}(B)$ is an IFGSCS in (X, τ) for every IFCS B of (Y, σ) .

Definition⁸ 2.22:[8] A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy regular weakly generalized continuous (IFRWG continuous in short) if $f^{-1}(A)$ is an IFRWGCS in (X, τ) for every IFCS A of (Y, σ) .

Definition⁷ 2.23: An IFTS (X, τ) is said to be an intuitionistic fuzzy ${}_{\text{rw}}T_{1/2}$ (IF ${}_{\text{rw}}T_{1/2}$ in short) space if every IFRWGCS in X is an IFCS in X .

Definition⁷ 2.24: An IFTS (X, τ) is said to be an intuitionistic fuzzy ${}_{\text{rwg}}T_{1/2}$ (IF ${}_{\text{rwg}}T_{1/2}$ in short) space if every IFRWGCS in X is an IFPCS in X .



3. INTUITIONISTIC FUZZY COMPLETELY REGULAR WEAKLY GENERALIZED CONTINUOUS MAPPINGS

In this section, we introduce intuitionistic fuzzy completely regular weakly generalized continuous mappings and studied some of its properties.

Definition 3.1: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy completely regular weakly generalized continuous (IFcRWG continuous in short) mapping if $f^{-1}(B)$ is an IFRCS in (X, τ) for every IFRWGCS B of (Y, σ) .

Theorem 3.2: Every IFcRWG continuous mapping is an IF continuous mapping but not conversely.

Proof: Assume that $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IFcRWG continuous mapping. Let B be an IFCS in Y . Then B is an IFRWGCS in Y . Since f is an IFcRWG continuous mapping, $f^{-1}(B)$ is an IFRCS in X . This implies $f^{-1}(B)$ is an IFCS in X . Hence the mapping f is an IF continuous mapping.

Example 3.3: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.2, 0.3), (0.7, 0.5) \rangle$, and $G_2 = \langle y, (0.2, 0.3), (0.7, 0.5) \rangle$. Then $\tau = \{ 0_-, G_1, 1_- \}$ and $\sigma = \{ 0_-, G_2, 1_- \}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IF continuous mapping. But f is not an IFcRWG continuous mapping, since $B = \langle y, (0.4, 0.5), (0.6, 0.5) \rangle$ is an IFRWGCS in Y but $f^{-1}(B) = \langle x, (0.4, 0.5), (0.6, 0.5) \rangle$ is not an IFRCS in X .

Theorem 3.4: Every IFcRWG continuous mapping is an IFG continuous mapping but not conversely.

Proof: Assume that $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IFcRWG continuous mapping. Let B be an IFCS in Y . This implies B is an IFRWGCS in Y . Since f is an IFcRWG continuous mapping, $f^{-1}(B)$ is an IFRCS in X . This implies $f^{-1}(B)$ is an IFGCS in X . Hence f is an IFG continuous mapping.

Example 3.5: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.3, 0.4), (0.7, 0.6) \rangle$, and $G_2 = \langle y, (0.4, 0.4), (0.6, 0.6) \rangle$. Then $\tau = \{ 0_-, G_1, 1_- \}$ and $\sigma = \{ 0_-, G_2, 1_- \}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFG continuous mapping. But f is not an IFcRWG continuous mapping, since $B = \langle y, (0.5, 0.4), (0.5, 0.5) \rangle$ is an IFRWGCS in Y but $f^{-1}(B) = \langle x, (0.5, 0.4), (0.5, 0.5) \rangle$ is not an IFRCS in X .

Theorem 3.6: Every IFcRWG continuous mapping is an IF α continuous mapping but not conversely.

Proof: Assume that $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IFcRWG continuous mapping. Let B be an IFCS in Y . This implies B is an IFRWGCS in Y . Since f is an IFcRWG continuous mapping, $f^{-1}(B)$ is an IFRCS in X . This implies $f^{-1}(B)$ is an IF α CS in X . Hence f is an IF α continuous mapping.

Example 3.7: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.5, 0.3), (0.2, 0.6) \rangle$ and $G_2 = \langle y, (0.5, 0.3), (0.2, 0.6) \rangle$. Then

$\tau = \{ 0_-, G_1, 1_- \}$ and $\sigma = \{ 0_-, G_2, 1_- \}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IF α continuous mapping. But f is not an IFcRWG continuous mapping, since $B = \langle y, (0.2, 0.4), (0.8, 0.5) \rangle$ is an IFRWGCS in Y but $f^{-1}(B) = \langle x, (0.2, 0.4), (0.8, 0.5) \rangle$ is not an IFRCS in X .

Theorem 3.8: Every IFcRWG continuous mapping is an IF α G continuous mapping but not conversely.

Proof: Assume that $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IFcRWG continuous mapping. Let B be an IFCS in Y . This implies B is an IFRWGCS in Y . Since f is an IFcRWG continuous mapping, $f^{-1}(B)$ is an IFRCS in X . This implies $f^{-1}(B)$ is an IF α GCS in X . Hence f is an IF α G continuous mapping.

Example 3.9: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.5, 0.7), (0.5, 0.3) \rangle$ and $G_2 = \langle y, (0.2, 0.3), (0.7, 0.7) \rangle$. Then we define $\tau = \{ 0_-, G_1, 1_- \}$ and $\sigma = \{ 0_-, G_2, 1_- \}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IF α G continuous mapping. But f is not an IFcRWG continuous mapping since $B = \langle y, (0.3, 0.1), (0.5, 0.8) \rangle$ is an IFRWGCS in Y but $f^{-1}(B) = \langle x, (0.3, 0.1), (0.5, 0.8) \rangle$ is not an IFRCS in X .

Theorem 3.10: Every IFcRWG continuous mapping is an IF γ continuous mapping but not conversely.

Proof: Assume that $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IFcRWG continuous mapping. Let B be an IFCS in Y . This implies B is an IFRWGCS in Y . Since f is an IFcRWG continuous mapping, $f^{-1}(B)$ is an IFRCS in X . This implies $f^{-1}(B)$ is an IF γ CS in X . Hence f is an IF γ continuous mapping.

Example 3.11: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.5, 0.3), (0.5, 0.7) \rangle$, $G_2 = \langle x, (0.3, 0.2), (0.7, 0.8) \rangle$ and $G_3 = \langle y, (0.3, 0.2), (0.7, 0.8) \rangle$. Then we define $\tau = \{ 0_-, G_1, G_2, 1_- \}$ and $\sigma = \{ 0_-, G_3, 1_- \}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IF γ continuous mapping. But f is not an IFcRWG continuous mapping since $B = \langle y, (0.4, 0.3), (0.3, 0.2) \rangle$ is an IFRWGCS in Y but $f^{-1}(B) = \langle x, (0.4, 0.3), (0.3, 0.2) \rangle$ is not an IFRCS in X .

Theorem 3.12: Every IFcRWG continuous mapping is an IFS continuous mapping but not conversely.

Proof: Assume that $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IFcRWG continuous mapping. Let B be an IFCS in Y . Then B is an IFRWGCS in Y . Since f is an IFcRWG continuous mapping, $f^{-1}(B)$ is an IFRCS in X . This implies $f^{-1}(B)$ is an IFS in X . Hence f is an IFS continuous mapping.

Example 3.13: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.4, 0.3), (0.6, 0.7) \rangle$ and $G_2 = \langle y, (0.5, 0.4), (0.5, 0.6) \rangle$. Then $\tau = \{ 0_-, G_1, 1_- \}$ and $\sigma = \{ 0_-, G_2, 1_- \}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFS continuous mapping. But f is not an IFcRWG continuous mapping since $B = \langle y, (0.6, 0.7), (0.3, 0.3) \rangle$ is an IFRWGCS in Y but $f^{-1}(B) = \langle x, (0.6, 0.7), (0.3, 0.3) \rangle$ is not an IFRCS in X .

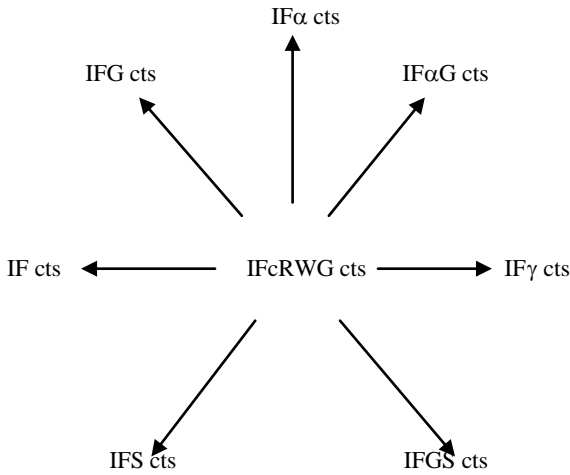


Fig.1 Relation between intuitionistic fuzzy completely regular weakly generalized continuous mappings and other existing intuitionistic fuzzy mappings

In this diagram by “A \longrightarrow B” we mean A implies B but not conversely.

None of them is reversible.

Theorem 3.14: Every IFcRWG continuous mapping is an IFGS continuous mapping but not conversely.

Proof: Assume that $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IFcRWG continuous mapping. Let B be an IFCS in Y. This implies B is an IFRWGCS in Y. Since f is an IFcRWG continuous mapping, $f^{-1}(B)$ is an IFRC in X. This implies $f^{-1}(B)$ is an IFGCS in X. Hence f is an IFGS continuous mapping.

Example 3.15: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.4, 0.3), (0.6, 0.7) \rangle$, $G_2 = \langle x, (0.7, 0.7), (0.3, 0.3) \rangle$ and $G_3 = \langle y, (0.5, 0.4), (0.5, 0.6) \rangle$. Then we define $\tau = \{ 0., G_1, G_2, 1. \}$ and $\sigma = \{ 0., G_3, 1. \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFGS continuous mapping. But f is not an IFcRWG continuous mapping since $B = \langle y, (0.4, 0.5), (0.5, 0.5) \rangle$ is an IFRWGCS in Y but $f^{-1}(B) = \langle x, (0.6, 0.8), (0.1, 0.1) \rangle$ is not an IFRC in X.

Theorem 3.16: Every IFcRWG continuous mapping is an IFRWG irresolute mapping.

Proof: Assume that $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IFcRWG continuous mapping. Let B be an IFRWGCS in Y. Since f is an IFcRWG continuous mapping, $f^{-1}(B)$ is an IFRC in X. This implies $f^{-1}(B)$ is an IFRWGCS in X. Hence f is an IFRWG irresolute mapping.

Theorem 3.17: A mapping $f : X \rightarrow Y$ is an IFcRWG continuous mapping if and only if the inverse image of each IFRWGOS in Y is an IFROS in X.

Proof: Necessity: Let A be an IFRWGOS in Y. This implies A^c is an IFRWGCS in Y. Since f is an IFcRWG continuous, $f^{-1}(A^c)$ is an IFRC in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFROS in X.

$f^{-1}(A^c)$ is an IFRC in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFROS in X.

Sufficiency: Let A be an IFRWGCS in Y. This implies A^c is an IFRWGOS in Y. By hypothesis $f^{-1}(A^c)$ is an IFROS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFRC in X. Hence f is an IFcRWG continuous mapping.

Theorem 3.18: Let $c(\alpha, \beta)$ be an IFP in an IFTS (X, τ) . A mapping $f : X \rightarrow Y$ is an IFcRWG continuous mapping if for every IFcRWGOS A in Y with $f(c(\alpha, \beta)) \in A$, there exists an IFROS B in X with $c(\alpha, \beta) \in B$ such that $f^{-1}(A)$ is IFD in B.

Proof : Let A be an IFRWGOS in Y and let $f(c(\alpha, \beta)) \in A$. Then there exists an IFROS B in X with $c(\alpha, \beta) \in B$ such that $cl(f^{-1}(A)) = B$. Since B is an IFROS, $cl(f^{-1}(A))$ is also an IFROS in X. Therefore, $int(cl(cl(f^{-1}(A)))) = cl(f^{-1}(A))$. That is $int(cl(f^{-1}(A))) = cl(f^{-1}(A))$. This implies $f^{-1}(A)$ is also an IFROS in X.

Theorem 3.19: If a mapping $f : X \rightarrow Y$ is an IFcRWG continuous mapping, then for every IFP $c(\alpha, \beta) \in X$ and for every IFN A of $f(c(\alpha, \beta))$, there exists an IFROS $B \subseteq X$ such that $c(\alpha, \beta) \in B \subseteq f^{-1}(A)$.

Proof: Let $c(\alpha, \beta) \in X$ and let A be an IFN of $f(c(\alpha, \beta))$. Then there exists an IFOS C in Y such that $f(c(\alpha, \beta)) \in C \subseteq A$. Since every IFOS is an IFRWGOS, C is an IFRWGOS in Y. Hence by hypothesis, $f^{-1}(C)$ is an IFROS in X and $c(\alpha, \beta) \in f^{-1}(C)$. Now, let $f^{-1}(C) = B$. Therefore $c(\alpha, \beta) \in B \subseteq f^{-1}(A)$.

Theorem 3.20: If a mapping $f : X \rightarrow Y$ is an IFcRWG continuous mapping, then for every IFP $c(\alpha, \beta) \in X$ and for every IFN A of $f(c(\alpha, \beta))$, there exists an IFROS $B \subseteq X$ such that $c(\alpha, \beta) \in B$ and $f(B) \subseteq A$.

Proof: Let $c(\alpha, \beta) \in X$ and let A be an IFN of $f(c(\alpha, \beta))$. Then there exists an IFOS C in Y such that $f(c(\alpha, \beta)) \in C \subseteq A$. Since every IFOS is an IFRWGOS, C is an IFRWGOS in Y. Hence by hypothesis, $f^{-1}(C)$ is an IFROS in X and $c(\alpha, \beta) \in f^{-1}(C)$. Now let $f^{-1}(C) = B$. Therefore $c(\alpha, \beta) \in B \subseteq f^{-1}(A)$. Thus $f(B) \subseteq f(f^{-1}(A)) \subseteq A$, which implies $f(B) \subseteq A$.

Theorem 3.21: A mapping $f : X \rightarrow Y$ is an IFcRWG continuous mapping then $int(cl(f^{-1}(int(B)))) \subseteq f^{-1}(B)$ for every IFS B in Y.

Proof: Let $B \subseteq Y$ be an IFS. Then, $int(B)$ is an IFOS in Y and hence an IFRWGOS in Y. By hypothesis, $f^{-1}(int(B))$ is an IFROS in X. Hence $int(cl(f^{-1}(int(B)))) = f^{-1}(int(B)) \subseteq f^{-1}(B)$.

Theorem 3.22: A mapping $f : X \rightarrow Y$ is an IFcRWG continuous mapping then the following are equivalent.

(i) for any IFRWGOS A in Y and for any IFP $c(\alpha, \beta) \in X$, if $f(c(\alpha, \beta)) \in A$ then $c(\alpha, \beta) \in int(f^{-1}(A))$.

(ii) for any IFRWGOS A in Y and for any $c(\alpha, \beta) \in X$, if $f(c(\alpha, \beta)) \in A$ then there exists an IFOS B such that $c(\alpha, \beta) \in B$ and $f(B) \subseteq A$.



Proof: (i) \Rightarrow (ii) Let $A \subseteq Y$ be an IFRWGOS and let $c(\alpha, \beta) \in X$. Let $f(c(\alpha, \beta)) \in A$. Then $c(\alpha, \beta) \in f^{-1}(A)$. (i) implies that $c(\alpha, \beta) \in \text{int}(f^{-1}(A))$, where $\text{int}(f^{-1}(A))$ is an IFOS in X . Let $B = \text{int}(f^{-1}(A))$. Since $\text{int}(f^{-1}(A)) \subseteq f^{-1}(A)$, $B \subseteq f^{-1}(A)$. Then, $f(B) \subseteq f(f^{-1}(A)) \subseteq A$.

(ii) \Rightarrow (i) Let $A \subseteq Y$ be an IFRWGOS and let $c(\alpha, \beta) \in X$. Suppose $f(c(\alpha, \beta)) \in A$, then by (ii) there exists an IFOS B in X such that $c(\alpha, \beta) \in B$ and $f(B) \subseteq A$. Now $B \subseteq f^{-1}(f(B)) \subseteq f^{-1}(A)$. That is $B = \text{int}(B) \subseteq \text{int}(f^{-1}(A))$. Therefore, $c(\alpha, \beta) \in \text{int}(f^{-1}(A))$.

Theorem 3.23: For any two IFcRWG continuous mappings f_1 and $f_2 : (X, \tau) \rightarrow (Y, \sigma)$, the function $(f_1, f_2) : (X, \tau) \rightarrow (Y \times Y, \sigma \times \sigma)$ is also an IFcRWG continuous mapping where $(f_1, f_2)(x) = (f_1(x), f_2(x))$ for every $x \in X$.

Proof : Let $A \times B$ be an IFRWGOS in $Y \times Y$. Then $(f_1, f_2)^{-1}(A \times B)(x) = (A \times B)(f_1(x), f_2(x)) = \langle (x, \min(\mu_A(f_1(x)), \mu_B(f_2(x))), \max(\nu_A(f_1(x)), \nu_B(f_2(x)))) \rangle = \langle (x, \min(f_1^{-1}(\mu_A(x)), f_2^{-1}(\mu_B(x))), \max(f_1^{-1}(\nu_A(x)), f_2^{-1}(\nu_B(x)))) \rangle = f_1^{-1}(A) \cap f_2^{-1}(B)(x)$. Since f_1 and f_2 are IFcRWG continuous mappings, $f_1^{-1}(A)$ and $f_2^{-1}(B)$ are IFROSs in X . Since intersection of IFROSs is an IFROS, $f_1^{-1}(A) \cap f_2^{-1}(B)$ is an IFROS in X . Hence (f_1, f_2) is an IFcRWG continuous mappings.

Theorem 3.24: Let $f: X \rightarrow Y$ be a mapping. Then the following are equivalent.

- (i) f is an IFcRWG continuous mapping.
- (ii) $f^{-1}(B)$ is an IFROS in X for every IFRWGOS B in Y .
- (iii) for every IFP $c(\alpha, \beta) \in X$ and for every IFRWGOS B in Y such that $f(c(\alpha, \beta)) \in B$ there exists an IFROS A in X such that $c(\alpha, \beta) \in A$ and $f(A) \subseteq B$.

Proof : (i) \Rightarrow (ii) is obviously.
 (ii) \Rightarrow (iii) Let $c(\alpha, \beta) \in X$ and $B \subseteq Y$ such that $f(c(\alpha, \beta)) \in B$. This implies $c(\alpha, \beta) \in f^{-1}(B)$. Since B is an IFRWGOS in Y , by hypothesis $f^{-1}(B)$ is an IFROS in X . Let $A = f^{-1}(B)$. Then $c(\alpha, \beta) \in f^{-1}(f(c(\alpha, \beta))) \in f^{-1}(B) = A$. Therefore $c(\alpha, \beta) \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$. This implies $f(A) \subseteq B$.

(iii) \Rightarrow (i) Let $B \subseteq Y$ be an IFRWGOS. Let $c(\alpha, \beta) \in X$ and $f(c(\alpha, \beta)) \in B$. By hypothesis, there exists an IFROS C in X such that $c(\alpha, \beta) \in C$ and $f(C) \subseteq B$. This implies $C \subseteq f^{-1}(f(C)) \subseteq f^{-1}(B)$. Therefore, $c(\alpha, \beta) \in C \subseteq f^{-1}(B)$. That is $f^{-1}(B) = \bigcup_{c(\alpha, \beta) \in f^{-1}(B)} c(\alpha, \beta) \subseteq \bigcup_{c(\alpha, \beta) \in f^{-1}(B)} C \subseteq f^{-1}(B)$. This implies $f^{-1}(B) = \bigcup_{c(\alpha, \beta) \in f^{-1}(B)} C$. Since union of IFROSs is IFROS, $f^{-1}(B)$ is an IFROS in X . Hence f is an IFcRWG continuous mapping.

4. CONCLUSION

In this paper we have introduced intuitionistic fuzzy completely regular weakly generalized continuous mapping and studied some of its basic properties. Also we have studied the

relationship between intuitionistic fuzzy completely regular weakly generalized continuous mappings and some of the intuitionistic fuzzy continuous mappings that already exist

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