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Multi-objective Assignment Problem with Generalized Trapezoidal Fuzzy Numbers

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ABSTRACT
The aim of this paper is to study multi-objective assignment problem with imprecise costs, time and ineffectiveness instead of its precise information. Here, elements of cost matrix, consumed time matrix and ineffectiveness level matrix have been represented by generalized trapezoidal fuzzy numbers as it is suitable way to represent the impreciseness of values provided by the decision makers due to time pressure or limited information and poor information processing capabilities. A priority based fuzzy goal programming method has been developed for generalized trapezoidal fuzzy numbers and it is applied for multi-objective assignment problem. Euclidean distance function is used to identify the most appropriate priority structure of fuzzy goals among the different priorities of the fuzzy goals. An illustrative numerical example is provided to demonstrate the effectiveness of the proposed approach.

General Terms
Priority based fuzzy goal programming.

Keywords
Fuzzy sets; Generalized trapezoidal fuzzy numbers; Multi-objective assignment problem; Priority based fuzzy goal programming

1. INTRODUCTION
The assignment problem (AP) is a well studied tropic in combinatorial optimization. It has a well connection in production planning, telecommunication, VLSI design, economics etc., where it deals with the question how to set n assignee to m tasks in an injective way for which an optimal assignment can be made in the best possible way. Depending on the objectives one has to optimize different problems ranging from linear assignment problem to quadratic and higher dimensional AP. The linear assignment problem is a special type of linear programming problem where assignees are being assigned to perform tasks in one to one basis such that the assignment cost (or profit) is minimum (or maximum). The best assignee for the task is a perfect description of the assignment problem, where number of rows and columns are identical. In 1955, Kuhn proposed an algorithm for linear assignment problem known as Hungarian method [1]. It is used for solving single objective assignment problem in crisp environment. Linear programming method can also be used to solve single objective assignment problem. Basically, single-objective assignment problem deals with the problem of cost minimizing or time minimizing.

We often come in close contact with an assignment problem where, cost and time are jointly co-related. Geetha et al [2] first formulated cost-time assignment problem as the multi-criteria problem. However, in our real life, assigning tasks based on only cost and time cannot properly reflect the real situation as decision maker cannot ignore the importance of quality of tasks. Bao et al. [3] developed a multi-objective assignment problem (MOAP). They used 0-1 programming method to convert multi-objective assignment problem to single objective assignment problem.

However, assignment problem representing real-world situations involves a set of parameters whose values are assigned by decision makers (DMs). In the conventional approach, DMs are required to fix exact values to the said parameters. In that case, DMs do not precisely know the exact value of those parameters. If exact values are suggested these are only statistical inference from past data and their stability is doubtful, so the parameters of the problem are usually defined by the DM in an uncertain way. Therefore, the DMs can easily express their opinions based on fuzzily described parameters.

Lin and Wen [4] solved assignment problem with fuzzy interval costs by the labeling algorithm. They showed that assignment problem can usually be simplified into either a linear fractional programming problem or a bottleneck assignment problem. Chen [5] proposed a fuzzy assignment problem by assuming that all the individuals involved have same skills.

Tsai et al. [6] studied multi-objective decision making problem associated with cost, time and quality by fuzzy approach. Belacela et al. [7] studied multi-criterion fuzzy assignment problem. Majumder and Bhunia [8] discussed a generalized assignment problem with interval valued numbers via elitist genetic algorithm developed by interval valued fitness function. Kumar and Gupta [9] developed a solution method for fuzzy assignment problems and fuzzy travelling salesman problems with different membership functions with Yager's ranking index [10]. Emrouznejad et al. [11] developed an alternative formulation for the fuzzy assignment problem with fuzzy costs or fuzzy profits for each possible assignment based on data envelopment analysis. Haddad et al. [12] discussed two models for the generalized assignment problem in uncertain environment. They used novel hybrid algorithm using simulated annealing (SA) method and max-min fuzzy in order to obtain near optimal solution. Biswas and Pramanik [13] studied MOAP with fuzzy costs. They solved it by using Yager's ranking method after transforming MOAP to single-objective assignment problem.

Pramanik and Roy [14] studied multi-objective transportation model with normal trapezoidal fuzzy numbers based on priority based fuzzy goal programming. They solved it with different priority structure and determined the most satisfactory solution by using Euclidean distance function. In this paper, we propose a priority based fuzzy goal programming method for generalized trapezoidal fuzzy numbers (TrFNs). Euclidean distance function is used to identify the most appropriate priority structure of fuzzy goals among the different priorities of the fuzzy goals. An illustrative numerical example of multi-objective assignment problem with TrFNs is provided to demonstrate the effectiveness of the proposed approach.

Rest of the paper is organized in the following way: Section 2 describes the preliminaries of fuzzy sets required for the paper. Section 3 presents the formulation of priority based fuzzy goal programming with generalized trapezoidal fuzzy numbers. In Section 4 the formulation of multi-objective assignment problem has been provided. Section 5 provides the formulation of multi-objective assignment problem with generalized trapezoidal fuzzy numbers. Section 6 is devoted to present proposed priority based fuzzy goal programming method to fuzzy multi-objective assignment. Section 7 deals with the selection of appropriate priority structure using Euclidean distance function. Numerical example has been solved in Section 8 to show the efficiency of the proposed method. Finally, Section 9 presents the concluding remarks.

2. PRELIMINARIES OF FUZZY SETS

In 1965, Zadeh [15] introduced the concept of fuzzy sets.

2.1 Definition: Fuzzy set: A fuzzy set $\tilde{A}$ in a universe of discourse X is defined by $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x) \langle \mid x \in X\}$, where $\mu_{\tilde{A}}(x) : X \to [0, 1]$ is called the membership function of $\tilde{A}$ and $\mu_{\tilde{A}}(x)$ is the degree of membership to which $x \in \tilde{A}$.

2.2 Definition: Normal fuzzy set: A fuzzy set $\tilde{q}$ is said to normal if there exists a point x in X such that $\mu_{\tilde{q}}(x) = 1$. Otherwise, $\tilde{q}$ is said to be the subnormal fuzzy set.

2.3 Definition: Trapezoidal fuzzy number (TrFN): A trapezoidal fuzzy number $\tilde{q}$ is denoted by (q_1, q_2, q_3, q_4) where q_1, q_2, q_3, q_4 are real numbers and its membership function

$\mu_{\tilde{q}}(x)$ is defined by $\mu_{\tilde{q}}(x)$ =

$$\begin{cases} 0, & x \le q_1, \\ \mu_L(x) = \dfrac{x - q_1}{q_2 - q_1}, & q_2 \le x \le q_1, \\ 1, & q_2 \le x \le q_3, \\ \mu_U(x) = \dfrac{q_4 - x}{q_4 - q_3}, & q_3 \le x \le q_4, \\ 0, & x \ge q_4 \end{cases} \quad (1)$$

$\mu_{\tilde{q}}(x)$ satisfies the following conditions.

(i) $\mu_{\tilde{q}}(x)$ is a continuous mapping from $\Re$ to closed interval [0, 1]

(ii) $\mu_{\tilde{q}}(x) = 0$ for every $x \in (-\infty, q_1]$

(iii) $\mu_{\tilde{q}}(x)$ is strictly increasing and continuous on $[q_1, q_2]$

(iv) $\mu_{\tilde{q}}(x) = 1$ for every $x \in [q_2, q_3]$

(v) $\mu_{\tilde{q}}(x)$ is strictly decreasing and continuous on $[q_3, q_4]$

(vi) $\mu_{\tilde{q}}(x) = 0$ for every $x \in [q_4, \infty)$

2.4 Definition: Generalized trapezoidal fuzzy number: A generalized TrFN $\tilde{q}$ is denoted by (q_1, q_2, q_3, q_4) where q_1, q_2, q_3, q_4 and $w \in [0,1]$ are real numbers and its membership function is represented by $\mu_{\tilde{q}}(x)$ and it is defined by

$\mu_{\tilde{q}}(x)$ =

$$\begin{cases} 0, & x \le q_1, \\ \mu_L(x) = w\left(\dfrac{x - q_1}{q_2 - q_1}\right), & q_2 \le x \le q_1, \\ 1, & q_2 \le x \le q_3, \\ \mu_U(x) = w\left(\dfrac{q_4 - x}{q_4 - q_3}\right), & q_3 \le x \le q_4, \\ 0, & x \ge q_4 \end{cases} \quad (2)$$

2.4 Definition: The $\alpha-$cut set of a fuzzy set $\tilde{q}$ is a crisp set defined by $\tilde{q}_\alpha = \{x \in X / \mu_{\tilde{q}}(x) \ge \alpha\}$.

2.5 Arithmetic Operation:

The arithmetic operations [16] of addition, subtraction, multiplication, and division of generalized TrFNs are defined as follows:

2.5.1 Addition: Let $\tilde{A} = (a_1, a_2, a_3, a_4; w_1)$ and $\tilde{B} = (b_1, b_2, b_3, b_4; w_2)$ be two generalized TrFNs, then

$\tilde{A} + \tilde{B} = (a_1, a_2, a_3, a_4; w_1) + (b_1, b_2, b_3, b_4; w_2)$

$= (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; \min(w_1, w_2))$

2.5.2 Subtraction: $\tilde{A} - \tilde{B} = (a_1, a_2, a_3, a_4; w_1) - (b_1, b_2, b_3, b_4; w_2) = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1; \min(w_1, w_2))$

2.5.3 Multiplication: $\tilde{A}_1 \otimes \tilde{A}_2 = (a_1, a_2, a_3, a_4; w_1) \otimes (b_1, b_2, b_3, b_4; w_2)$
$= \{\min(a_1b_1, a_1b_4, a_4b_1, a_4b_4), \min(a_2b_2, a_2b_3, a_3b_2, a_3b_3), \max(a_2b_2, a_2b_3, a_3b_2, a_3b_3), \max(a_1b_1, a_1b_4, a_4b_1, a_4b_4); \min(w_1, w_2)\}$

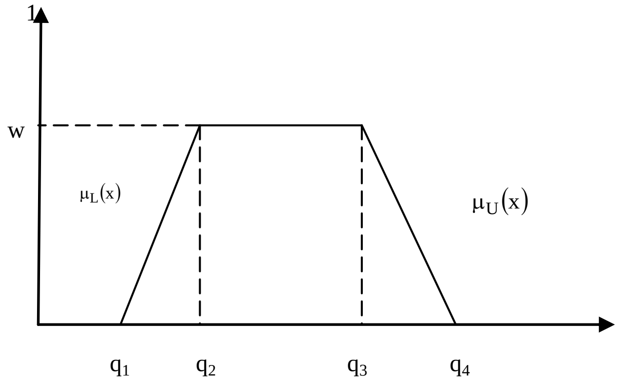

Figure1. A generalized trapezoidal fuzzy number
$\tilde{q} = (q_1, q_2, q_3, q_4)$

3. FORMULATION OF PRIORITY BASED FUZZY GOAL PROGRAMMING WITH GENERALIZED TRFNS

We consider the following multi-objective problem with TrFNs.

Minimize $\tilde{\phi}_k(\bar{x}) = (\tilde{c}_1\bar{x}, \tilde{c}_2\bar{x}, ..., \tilde{c}_k\bar{x})^T$ (3)

subject to $\bar{x} \in X = \{\bar{x} \in \Re^n / \tilde{A}\bar{x} * \tilde{b}, \bar{x} \geq \bar{0}\}$ (4)

where, $\tilde{c}_k$ (k = 1, 2,…, K) denotes n-dimensional row vector, $\tilde{b}$ denotes an m- dimensional column vector, $\tilde{A}$ represents an $m \times n$ matrix and $\bar{x} = (x_1, x_2, ..., x_n)^T$. $\tilde{c}$, $\tilde{b}$, and $\tilde{A}$ are all generalized trapezoidal TrFNs. Here, the symbol $*$ denotes the symbol $\geq, =,$ and $\leq$ respectively. Now we consider the problem represented by (3) has fuzzy coefficients with possibilistic distribution. We assume that $\bar{X}$ be a solution of (11), where, $\alpha \in [0,1]$ represents the possibility at which all the fuzzy co-efficient is feasible.

Let ${}^{\alpha}(\tilde{q})$ be the $\alpha-$cut of a fuzzy number $\tilde{q}$ defined by ${}^{\alpha}(\tilde{q}) = \{q \in Supp(\tilde{q}) / \mu_{\tilde{q}}(q) \geq \alpha, \alpha \in [0,1]\}$ where Supp($\tilde{q}$) is the support of $\tilde{q}$. (5)

Let ${}^{\alpha}(\tilde{q})^L$ and ${}^{\alpha}(\tilde{q})^U$ be the lower and upper bound of $\alpha-$cut of $\tilde{q}$ such that ${}^{\alpha}(\tilde{q})^L \leq {}^{\alpha}(q) \leq {}^{\alpha}(\tilde{q})^U$. (6)

For maximization type objective function, for any pre-assigned value of α, $\tilde{\phi}_k(\bar{x})$ (k = 1, 2,…, K) can be replaced by the upper bound of its $\alpha-$cut that is

$${}^{\alpha}(\tilde{\phi}_k(\bar{x}))^U = \sum_{j=1}^{n} {}^{\alpha}(\tilde{c}_{kj})^U x_j \quad (7)$$

Similarly, for minimization type of objective function $\tilde{\phi}_k(\bar{x})$ (k = 1, 2,…, K), we consider the lower bound of its $\alpha-$cut i.e.

$${}^{\alpha}(\tilde{\phi}_k(\bar{x}))^L = \sum_{j=1}^{n} {}^{\alpha}(\tilde{c}_{kj})^L x_j \quad (8)$$

Now we can rewrite for the inequality constraints as:

$$\sum_{j=1}^{n} \tilde{A}_{ij} x_j \geq \tilde{b}_j, \ i = 1, 2, \ldots m_1 \quad (9)$$

and $\sum_{j=1}^{n} \tilde{A}_{ij} x_j \leq \tilde{b}_j$, i = $m_1 + 1, \ldots, m_2$ (10)

by the following way $\sum_{j=1}^{n} {}^{\alpha}(\tilde{A}_{ij})^U \geq {}^{\alpha}(\tilde{b})^L$, i = 1, 2,… m_1 (11)

$$\sum_{j=1}^{n} {}^{\alpha}(\tilde{A}_{ij})^L \leq {}^{\alpha}(\tilde{b})^U, \ i = m_1 + 1, \ldots, m_2. \quad (12)$$

For fuzzy equality constraints

$$\sum_{j=1}^{n} \tilde{A}_{ij} x_j = \tilde{b}_j, \ i = m_2 + 1, ..., m \quad (13)$$

can be represented by two equivalent constraints

$$\sum_{j=1}^{n} {}^{\alpha}(\tilde{A}_{ij})^L \leq {}^{\alpha}(\tilde{b})^U \quad (14)$$

and $\sum_{j=1}^{n} {}^{\alpha}(\tilde{A}_{ij})^U \geq {}^{\alpha}(\tilde{b})^L$ (15)

For proof of the equivalency of (13) with (14) and (15), see Lee and Li [17]

Therefore, for any pre-assigned value of α, the problem represented by (3) can be transformed into an equivalent form as:

Minimize ${}^{\alpha}(\tilde{\phi}_k)^L = \sum_{j=1}^{n} {}^{\alpha}(\tilde{c}_{kj})^L x_j$, k = 1, 2,…, K (16)

Subject to $\sum_{j=1}^{n} {}^{\alpha}(\tilde{A}_{ij})^U \geq {}^{\alpha}(\tilde{b})^L$, i = 1, 2,…, m_1, m_2+1, …, m, (17)

$\sum_{j=1}^{n} {}^{\alpha}(\tilde{A}_{ij})^L \leq {}^{\alpha}(\tilde{b})^U$, i = m_1+1, …, m_2, m_2+1, …, m, (18)

$x_j \geq 0, \ j = 1, 2, ..., n.$ (19)

For simplicity, we denote the system of constraints (17), (18) and (19) as ψ. We notice that, for any prescribed value of α, the problem (16) reduces to a deterministic multi-objective linear programming problem. It can be solved by applying fuzzy goal programming technique.

Now we construct the membership function for minimization type objective function as:

$${}^{\alpha}\mu_k\left({}^{\alpha}(\tilde{\phi}_k)^L\right) = \frac{\left[{}^{\alpha}(\tilde{\phi}_k)^- - \sum_{j=1}^{n} {}^{\alpha}(\tilde{c}_{kj})^L x_j\right]}{\left[{}^{\alpha}(\tilde{\phi}_k)^- - {}^{\alpha}(\tilde{\phi}_k)^0\right]}, \ k = 1, 2, \ldots, K \quad (20)$$

Where, the aspired level ${}^{\alpha}(\phi_k)^0$ and highest acceptable level ${}^{\alpha}(\phi_k)^-$ are ideal and anti- ideal solutions, respectively. It can be obtained by solving the following problem:

$${}^{\alpha}(\tilde{\phi}_k)^0 = \min_{X \in \psi} \sum_{j=1}^{n} {}^{\alpha}(\tilde{c}_{kj})^L x_j, \quad (21)$$

$${}^{\alpha}(\tilde{\phi}_k)^- = \max_{X \in \psi} \sum_{j=1}^{n} {}^{\alpha}(\tilde{c}_{kj})^U x_j, \ k = 1, 2, \ldots, K. \quad (22)$$

Similarly, for the maximization type objective function, we can also determine ideal and anti-ideal solutions. To deal with the problem (3), we assume that all the fuzzy coefficients are generalized trapezoidal fuzzy numbers. Then from definition 2.4, the $\alpha-$cut representation of generalized trapezoidal fuzzy number $\tilde{q} = (q_1, q_2, q_3, q_4; w)$, we can write it by following interval

$$^{\alpha}(\tilde{q}) = \left[{}^{\alpha}(\tilde{q})^{L}, {}^{\alpha}(\tilde{q})^{U}\right] = \left[(q_2 - q_1)\alpha/w + q_1, q_4 - (q_4 - q_3)\alpha/w\right]$$

(23) Now, for a prescribed value of α, the fuzzy goal programming model of the problem under a pre-emptive priority structure [18] can be presented as

$$\text{Minimize } \tilde{\xi} = \left[P_1(d^-), P_2(d^-), \ldots, P_s(d^-), \ldots, P_S(d^-)\right], \quad (24)$$

subject to
$$\frac{\left[{}^{\alpha}(\tilde{\phi}_k)^{-} - \sum_{j=1}^{n} {}^{\alpha}(\tilde{c}_{kj})^{L} x_j\right]}{\left[{}^{\alpha}(\tilde{\phi}_k)^{-} - {}^{\alpha}(\tilde{\phi}_k)^{o}\right]} + d_k^- = 1, \; k = 1, 2, \ldots, K,$$

$$\sum_{j=1}^{n} {}^{\alpha}(\tilde{A}_{ij})^{U} \geq {}^{\alpha}(\tilde{b})^{L}, \; i = 1, 2, \ldots m_1, m_2+1, \ldots, m,$$

$$\sum_{j=1}^{n} {}^{\alpha}(\tilde{A}_{ij})^{L} \leq {}^{\alpha}(\tilde{b})^{U}, \; i = m_1+1, \ldots, m_2, m_2+1, \ldots, m,$$

$$x_j \geq 0, \; j = 1, 2, \ldots, n.$$

$$d_k^- \geq 0, k = 1, 2, \ldots, K. \quad (25)$$

Using the interval expression, the problem (25) can be written as:

$$\text{Minimize } \tilde{\xi} = \left[P_1(d^-), P_2(d^-), \ldots, P_s(d^-), \ldots, P_S(d^-)\right],$$

subject to,

$$\frac{\left[{}^{\alpha}(\tilde{\phi}_k)^{-} - \sum_{j=1}^{n}\left[c_{kj}^{1} + (c_{kj}^{2} - c_{kj}^{1})\,\alpha/w_{kj}\right]x_j\right]}{\left[{}^{\alpha}(\tilde{\phi}_k)^{-} - {}^{\alpha}(\tilde{\phi}_k)^{o}\right]} + d_k^- = 1, \; k = 1,$$

2,…,K.

$$\left[A_{ij}^{1} + (A_{ij}^{2} - A_{ij}^{1})\,\alpha/w_{ij}\right]x_j \geq b_i^{4} + (b_i^{4} - b_i^{3})\,\alpha/w_{ij}, \quad i =$$

$$m_1+1, \ldots, m_2, m_2+1, \ldots, m,$$

$$x_j \geq 0, \; j = 1, 2, \ldots, n.$$

4. FORMULATION OF MULTI-OBJECTIVE ASSIGNMENT PROBLEM (MOAP)

For a single objective assignment problem (AP), n jobs are to be performed by n persons depending on their efficiency to do the job in one to one basis such that the assignment cost is minimal. Now if the objective of an AP is to minimize operation cost, operation time, and bad quality, etc then we treat this type of problem as a multi objective AP. Now considering this type of criterion which has an imprecise value, presented as a trapezoidal fuzzy number, we formulate a fuzzy multi-objective assignment problem (FMOAP) as:

$$\text{Min } \phi_k(\bar{x}) = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}^{k} x_{ij}, \quad k = 1, 2, \ldots, K. \quad (26)$$

$$\text{subject to } \sum_{i=1}^{n} x_{ij} = 1, \; j = 1, 2, \ldots, n. \quad (27)$$

$$\sum_{i=1}^{n} x_{ij} = 1, \; i = 1, 2, \ldots, n \quad (28)$$

$x_{ij} \in [0, 1]$, where x_{ij} =

$$\begin{cases} 1, & \text{if } i\text{-th person is assigned to } j\text{-th job.} \\ 0, & \text{otherwise.} \end{cases} \quad (29)$$

5. MULTI-OBJECTIVE ASSIGNMENT PROBLEM WITH GENERALIZED TRAPEZOIDAL FUZZY NUMBERS

In this section we discuss a FMOAP defined in (26) where, all the coefficients have been considered as generalized trapezoidal fuzzy numbers. Therefore, to keep impreciseness, the trapezoidal fuzzy number $\left(c_{ij}^{k(1)}, c_{ij}^{k(2)}, c_{ij}^{k(3)}, c_{ij}^{k(4)}; w_{ij}\right)$ can be nimize $\tilde{\phi}_k(\bar{x}) = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(c_{ij}^{k(1)}, c_{ij}^{k(2)}, c_{ij}^{k(3)}, c_{ij}^{k(4)}; w_{ij}\right)x_{ij}$,

k=1,2,...K (30)

$$\text{subject to } \sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, 2, \ldots, n. \quad (31)$$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad i = 1, 2, \ldots, n. \quad (32)$$

$x_{ij} \in [0, 1]$, where x_{ij} =

$$\begin{cases} 1, & \text{if } i\text{-th person is assigned to } j\text{-th job.} \\ 0, & \text{otherwise.} \end{cases} \quad (33)$$

Then the problem can be reduced to,

$$\text{Minimize } \tilde{\phi}_k(\bar{x}) = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(c_{ij}^{k(1)} + \left(c_{ij}^{k(2)} - c_{ij}^{k(1)}\right)\alpha/w\right)x_{ij}$$

k = 1, 2,…, K.

$$\text{subject to } \sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, 2, \ldots, n \quad (34)$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \ldots, n. \quad (35)$$

6. PRIORITY BASED FUZZY GOAL PROGRAMMING FORMULATION OF MOAP WITH FUZZY PARAMETERS

With the help of equation (20), we obtain the membership function for minimization of objective function as:

$$^{\alpha}\mu_k(\tilde{\phi}_k) = \frac{\left[{}^{\alpha}(\tilde{\phi}_k)^{-} - \sum_{i=1}^{n}\sum_{j=1}^{n}\left[c_{kj}^{1} + (c_{kj}^{2} - c_{kj}^{1})\,\alpha/w_{kj}\right]x_j\right]}{\left[{}^{\alpha}(\tilde{\phi}_k)^{-} - {}^{\alpha}(\tilde{\phi}_k)^{0}\right]} \quad (36)$$

Where, the aspired level ${}^{\alpha}(\tilde{\phi}_k)^{0}$ is ideal solution which can be obtained by solving the following problem

$$^{\alpha}(\tilde{\phi}_k)^{0} = \min_{\bar{x} \in s} \sum_{i=1}^{n}\sum_{j=1}^{n}\left[c_{kj}^{k(1)} + \left(c_{kj}^{k(2)} - c_{kj}^{k(1)}\right)\alpha/w_{kj}\right]x_{ij}, \text{ for k}$$

=1, 2,…, K
(37)

and the highest acceptable level ${}^{\alpha}(\tilde{\phi}_k)^{-}$ are considered as an anti- ideal solution that can be obtained similarly by solving the following problem

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$$^{\alpha}\left(\tilde{\phi}_k\right)^{-} = \underset{\bar{x}\in s}{\mathrm{Max}} \sum_{i=1}^{n}\sum_{j=1}^{n}\left[c_{kj}^{k(4)} - \left(c_{kj}^{k(4)} - c_{kj}^{k(3)}\right) \alpha \big/ w_{kj}\right] x_{ij}\,,$$

for k =1, 2,…, K (38)

Now the fuzzy goal programming model of the given problem under the mini-sum goal programming (GP), we can formulate for any pre-assigned value of α as the following way:

Minimize $\tilde{\xi} = \left[P_1(d^-), P_2(d^-), ..., P_s(d^-), ..., P_S(d^-)\right]$,

$$\frac{\left[{}^{\alpha}\left(\tilde{\phi}_k\right)^{-} - \sum_{j=1}^{n}\left[c_{kj}^{1} + (c_{kj}^{2} - c_{kj}^{1})\ \alpha \big/ w_{kj}\right] x_j\right]}{\left[{}^{\alpha}\left(\tilde{\phi}_k\right)^{-} - {}^{\alpha}\left(\tilde{\phi}_k\right)^{0}\right]} + D_k^{-} = 1 \qquad (39)$$

$d_k^{-} \geq 0$, $k = 1, 2, ..., K$. $x_{ij} \in [0,1]$ for i= 1, 2, …, n and j= 1, 2, …, n. Here $\bar{\xi}$ is the vector of s-th priority achievement functions. D_k^{-} is connected with k-th goal as a negative deviational variable. $P_i(d^-)$ is a linear function of the weighted negative deviational variables. Here $P_s(d^-)$ = $\sum_{i=1}^{K} w_{sk}^{-} d_{sk}^{-}$ (k= 1, 2, …, K; S ≤ K. For simplicity, we replace d_{sk}^{-} by d_k^{-} at the s-th priority level. W_{sk}^{-} is the numerical weight associated with d_{sk}^{-} that represent the weight of importance for achieving the aspired level of k-th goal with respect to other goals, which are grouped together at the s-th priority level. This weight can be determined as:

$$W_{sk}^{-} = \frac{1}{\left[{}^{\alpha}\left(\tilde{\phi}_k\right)^{-} - {}^{\alpha}\left(\tilde{\phi}_k\right)^{0}\right]_s}\text{, for k = 1, 2,…, K. (40)}$$

Where $\left[{}^{\alpha}\left(\tilde{\phi}_k\right)^{-} - {}^{\alpha}\left(\tilde{\phi}_k\right)^{0}\right]_s$ is the symbolic representation of $\left[{}^{\alpha}\left(\tilde{\phi}_k\right)^{-} - {}^{\alpha}\left(\tilde{\phi}_k\right)^{0}\right]$ at the s-th priority level.

In preemptive priority fuzzy goal programming, P_s denotes the s-th priority which is preferred to next priority level P_{s+1} for any possible imposed weight on P_{s+1}. Thus the relationships among the priorities are $P_1 >> P_2 >> …. >> P_s >> … >> P_S$.

This indicates that the higher priority level is regarded as infinitely more important than lower priority level. This process will be continued till the last priority is considered. It is to be noted that not more than five priority level should be used in the priority based fuzzy goal programming as it would be unusual to find a practical situation with more than five mutually non comparable priorities.

7. SELECTION OF APPROPRIATE PRIORITY STRUCTURE USING EUCLIDEAN DISTANCE FUNCTION

The concept of distance function for group decision analysis was introduced by Yu [19]. In this paper Euclidean distance function is used for identifying proper priority structure for priority based fuzzy goal programming model [14]. The Euclidean distance function can be defined as:

$$L_2 = \left[\sum_{n=1}^{N}\left(1 - \mu_{\phi_n}^{t}\right)\right]^{1/2} \qquad (41)$$

Where $\mu_{\phi_n}^{t}$ represents the achieved membership value of the n-th goal under the t-th priority structure of the goals. Now the minimal Euclidean distance function for those achieved solution would correspond the most satisfying solution. Here it should be noted that the solution which is closest to the ideal point must correspond to $\min_{1,2,...,S} L_s = L_M$ where, $1 \leq M \leq S$.

It indicates the M-th priority structure can be identified as an appropriate priority structure to achieve the desired result.

8. ILLUSTRATIVE EXAMPLE

A manufacturing company manufactures machinery parts with four different machines for construction purposes. The company official has been given an order to execute the four different Tasks. To complete these tasks with these four machines, they have imprecise information about cost, time and ineffectiveness of each machine to perform the task (out of four) in one to one basis. Here these imprecise information (treated as TrFNs) about cost, time and ineffectiveness of each machine with respect to each Task are given in Table 1. Determine the optimal assignment.

The problem can be re-written as:

Minimize $\tilde{\phi}_1(\bar{x})$= (4,6,7,9; .833) x_{11} + (3,5,7,10; .933) x_{12} + (6,7,10,12; .891) x_{13} + (3,4,6,9; .822) x_{14} + (2,3,5,7; .947) x_{21} + (5,7,8,11; .848) x_{22} + (5,6,7,10; .800) x_{23} + (4,7,9,11; .988) x_{24} + (6,8,10,12; .878) x_{31} + (5,7,12,14; .900) x_{32} + (6,7,9,10; .867) x_{33} + (4,5,7,9; .913) x_{34} + (3,7,10,12; .911) x_{41} + (6,7,10,12; .864) x_{42} + (7,10,11,13; .790) x_{43} + (5,7,10,14; .814) x_{44};

Minimize $\tilde{\phi}_2(\bar{x})$= (7,9,11,13; .944) x_{11} +(6, 9, 10, 12; .818) x_{12} + (9, 10, 11, 12; .708) x_{13} + (8, 11, 13, 15; .761) x_{14} + (6, 7, 10, 12; .953) x_{21} +(9, 12, 14, 17; .792) x_{22} + (7, 8, 10, 11; .887) x_{23} + (6, 8, 12, 13; .901) x_{24} + (3, 4, 5, 7; .938) x_{31} + (4, 5, 7, 9; .946) x_{32} + (6, 7, 8, 11; .826) x_{33} + (3, 4, 6, 7; .901) x_{34} + (4, 6, 8, 10; .857) x_{41} + (5, 7, 8, 10; .804) x_{42} + (4, 5, 7, 8; .867) x_{43} + (5, 9, 11, 15; .857) x_{44};

Minimize $\tilde{\phi}_3(\bar{x})$= (0.15,0.16,0.19,0.21; .888) x_{11} + (0.10,0.11,0.13,0.14; .778) x_{12} + (0.14,0.16,0.18,0.20; .714) x_{13} + (0.05,0.07,0.09,0.11; .667) x_{14} + (0.09,0.12,0.15,0.18; .778) x_{21} + (0.14,0.16,0.18,0.20; .750) x_{22} + (0.20,0.21,0.23,0.25; .888) x_{23} + (0.15,0.18,0.22,0.25; .875) x_{24} + (0.18,0.20,0.22,0.24;

.667) x_{31} + (0.13,0.15,0.17,0.19; .778) x_{32} + (0.20,0.22,0.24,0.27; .667) x_{33} + (0.15,0.16,0.18,0.20; .625) x_{34} + (0.15,0.18,0.20,0.22; .750) x_{41} + (0.19,0.21,0.23,0.25; .888) x_{42} + (0.12,0.13,0.14,0.15; .778) x_{43} + (0.10,0.14,0.16,0.18; .750) x_{44};

subject to,

$x_{11}+x_{12}+x_{13}+x_{14}=1$; $x_{11}+x_{21}+x_{31}+x_{41}=1$,
$x_{21}+x_{22}+x_{23}+x_{24}=1$; $x_{12}+x_{22}+x_{32}+x_{42}=1$,
$x_{31}+x_{32}+x_{33}+x_{34}=1$; $x_{13}+x_{23}+x_{33}+x_{43}=1$,
$x_{41}+x_{42}+x_{43}+x_{44}=1$; $x_{14}+x_{24}+x_{34}+x_{44}=1$,
where, $x_{ij}\in\{0,1\}$. For i= 1, 2, 3, 4 and j= 1, 2, 3, 4. (42)

Now we replace all generalized fuzzy coefficient of the objective function by their corresponding $\alpha-$ cuts. Then the problem can be transformed into the following problem:

Minimize $^{\alpha}\left(\tilde{\phi}_1(\bar{x})\right)^L$ = (4 + 2.400 α) x_{11} +(3 + 2.144 α) x_{12} + (6 + 1.122 α) x_{13} + (3 + 1.216 α) x_{14} + (2 +1.055 α) x_{21} +(5 + 2.358 α) x_{22} + (5 + 1.25 α) x_{23} + (4 + 3.036 α) x_{24} + (6 + 2.278 α) x_{31} + (5 + 2.222 α) x_{32} + (6 +1.153 α) x_{33} + (4 + 1.095 α) x_{34} + (3 + 4.390 α) x_{41} + (6 + 1.157 α) x_{42} + (7 + 3.797 α) x_{43} + (5 + 2.457 α) x_{44};

Minimize $^{\alpha}\left(\tilde{\phi}_2(\bar{x})\right)^L$ = (7 + 2.119 α) x_{11} +(6 + 3.667 α) x_{12} + (9 + 1.412 α) x_{13} + (8 + 3.942 α) x_{14} + (6 + 1.049 α) x_{21} +(9 + 3.788 α) x_{22} + (7 +1.127 α) x_{23} + (6 + 2.220 α) x_{24} + (3 + 1.066 α) x_{31} + (4 + 1.057 α) x_{32} + (6 + 1.210 α) x_{33} + (3 + 1.110 α) x_{34} + (4 + 2.334 α) x_{41} + (5 + 2.487 α) x_{42} + (4 + 1.153 α) x_{43} + (5 + 4.667 α) x_{44};

Minimize $^{\alpha}\left(\tilde{\phi}_3(\bar{x})\right)^L$ = (.15 + .0111 α) x_{11} +(.10 + .0128 α) x_{12} + (.14 + .0280 α) x_{13} + (.05 + .0300 α) x_{14} + (.09 + .0386 α) x_{21} +(.14 + .0267 α) x_{22} + (.20 + .0112 α) x_{23} + (.15 + .0343 α) x_{24} + (.18 + .0300 α) x_{31} + (.13 + .0257 α) x_{32} + (.20 + .0300 α) x_{33} + (.15 + .0160 α) x_{34} + (.15 + .0400 α) x_{41} + (.19 + .0225 α) x_{42} + (.12 + .0129 α) x_{43} + (.10 + .0533 α) x_{44};

subject to,

$x_{11}+x_{12}+x_{13}+x_{14}=1$; $x_{11}+x_{21}+x_{31}+x_{41}=1$,
$x_{21}+x_{22}+x_{23}+x_{24}=1$; $x_{12}+x_{22}+x_{32}+x_{42}=1$,
$x_{31}+x_{32}+x_{33}+x_{34}=1$; $x_{13}+x_{23}+x_{33}+x_{43}=1$,
$x_{41}+x_{42}+x_{43}+x_{44}=1$; $x_{14}+x_{24}+x_{34}+x_{44}=1$,
where, $x_{ij}\in\{0,1\}$.
For i= 1, 2, 3, 4 and j= 1, 2, 3, 4. (43)

For different values of α we have some result, listed in the table- by the following procedure discussed in section 3.

We have for α =.5, $^{\alpha}\left(\tilde{\phi}_1\right)^{-}$ = 44.6567, $^{\alpha}\left(\tilde{\phi}_1\right)^{0}$ =19.2905, $^{\alpha}\left(\tilde{\phi}_2\right)^{a}$ = 48.8970, $^{\alpha}\left(\tilde{\phi}_2\right)^{0}$ = 22.4895, $^{\alpha}\left(\tilde{\phi}_3\right)^{-}$ = 0.9178, $^{\alpha}\left(\tilde{\phi}_3\right)^{0}$ = .4435.

Then the membership function corresponding to the objective function can be constructed

$$^{\alpha}\mu_1\left(\tilde{\phi}_1(\bar{x})\right)=\frac{\left[44.6567-{}^{\alpha}\left(\tilde{\phi}_1(\bar{x})\right)^L\right]}{[44.6567-19.2905]}=\frac{44.6567-{}^{\alpha}\left(\tilde{\phi}_1(\bar{x})\right)^L}{25.3662}$$
(44)

$$^{\alpha}\mu_2\left(\tilde{\phi}_2(\bar{x})\right)=\frac{\left[48.8970-{}^{\alpha}\left(\tilde{\phi}_2(\bar{x})\right)^L\right]}{[48.8970-22.4895]}=\frac{22.4895-{}^{\alpha}\left(\tilde{\phi}_2(\bar{x})\right)^L}{26.4075}$$
(45)

$$^{\alpha}\mu\left(\tilde{\phi}_3(\bar{x})\right)=\frac{\left[0.9178-{}^{\alpha}\left(\tilde{\phi}_3(\bar{x})\right)^L\right]}{[0.9178-.4435]}=\frac{0.9178-{}^{\alpha}\left(\tilde{\phi}_3(\bar{x})\right)^L}{.4743}$$
(46)

Then the priority based fuzzy goal programming for multi-objective assignment problem can be written as

$$\bar{\xi} = [\, P_1(d^-), P_1(d^-),..., P_s(d^-),..., P_S(d^-)\,]$$

subject to $\frac{44.6567-{}^{\alpha}\left(\tilde{\phi}_1(\bar{x})\right)^L}{25.3662}+d_1^-=1$,

$\frac{22.4895-{}^{\alpha}\left(\tilde{\phi}_2(\bar{x})\right)^L}{26.4075}+d_2^-=1$,

$\frac{0.9178-{}^{\alpha}\left(\tilde{\phi}_3(\bar{x})\right)^L}{.4743}+d_3^-=1$,

subject to,

$x_{11}+x_{12}+x_{13}+x_{14}=1$; $x_{11}+x_{21}+x_{31}+x_{41}=1$,
$x_{21}+x_{22}+x_{23}+x_{24}=1$; $x_{12}+x_{22}+x_{32}+x_{42}=1$,
$x_{31}+x_{32}+x_{33}+x_{34}=1$; $x_{13}+x_{23}+x_{33}+x_{43}=1$,
$x_{41}+x_{42}+x_{43}+x_{44}=1$; $x_{14}+x_{24}+x_{34}+x_{44}=1$,
$d_k^-\geq 0$ for (t = 1, 2, 3)
where, $x_{ij}\in\{0,1\}$. For i= 1, 2, 3, 4 and j= 1, 2, 3, 4.

In this example we consider three priorities factor P_i (i = 1, 2, 3) for achievement of aspired levels of the assigned fuzzy goals. Under different priority structure using LINGO 11.0, we solve this problem with our proposed priority based FGP of MOAP with fuzzy parameter, discussed in section 6 and the results are provided in Table 2.

From Table 2 we see that the minimum Euclidean distance is 0.1230585. This result reflects that the priority structure under the execution 2 and 5 is appropriate one to obtain the most satisfactory solution. Thus the optimal assignment schedule can be written as x^*_{12}= x^*_{21}=x^*_{34}=x^*_{43}=1 and others variables are zeros.

Table 1. A three objective Assignment problem

C, T, I/ M_i	Task-A	Task-B	Task-C	Task-D
M_1	(4,6,7,9; .833) (7,9,11,13; .944) (0.15,0.16,0.19,0.21; .888)	(3,5,7,10; .933) (6,9,10,12; .818) (0.10,0.11,0.13,0.14; .778)	(6,7,10,12; .891) (9,10,11,12; .708) (0.14,0.16,0.18,0.20; .714)	(3,4,6,9; .822) (8,11,13,15; .761) (0.05,0.07,0.09,0.11; .667)
M_2	(2,3,5,7; .947) (6,7,10,12; .953) (0.09,0.12,0.15,0.18; .778)	(5,7,8,11; .848) (9,12,14,17; .792) (0.14,0.16,0.18,0.20; .750)	(5,6,7,10; .800) (7,8,10,11; .887) (0.20,0.21,0.23,0.25; .888)	(4,7,9,11; .988) (6,8,12,13; .901) (0.15,0.18,0.22,0.25; .875)
M_3	(6,8,10,12; .878) (3,4,5,7; .938) (0.18,0.20,0.22,0.24; .667)	(5,7,12,14; .900) (4,5,7,9; .946) (0.13,0.15,0.17,0.19; .778)	(6,7,9,10; .867) (6,7,8,11; .826) (0.20,0.22,0.24,0.27; .667)	(4,5,7,9; .913) (3,4,6,7; .901) (0.15,0.16,0.18,0.20; .625)
M_4	(3,7,10,12; .911) (4,6,8,10; .857) (0.15,0.18,0.20,0.22; .750)	(6,7,10,12; .864) (5,7,8,10; .804) (0.19,0.21,0.23,0.25; .888)	(7,10,11,13; .790) (4,5,7,8;.867) (0.12,0.13,0.14,0.15; .778)	(5,7,10,14; .814) (5,9,11,15; .857) (0.10,0.14,0.16,0.18; .750)

Note: Here, C indicates cost, T indicates time, I indicates ineffectiveness of the machine, M_i presents i-th machine.

Table 2. Sensitivity analysis for solutions with different priority structure

Execution number	Priority Structure	Membership values	Euclidean Distance D_2	Min of F_1, F_2, F_3	Optimal solution
1	[P(d_1^- /25.3662+ d_2^- /26.4075 + d_3^- /.4743)]	0.9269 0.8822 0.9999	0.1386490	21.1450 25.6005 0.4435	$x_{14} = x_{21} = x_{32} = x_{43} = 1$; other x_{ij}'s are zero
2	[P(d_1^- /25.3662+ d_3^- /.4743, P(d_2^- /26.4075)]	0.9269 0.8822 0.9999	0.1386490	21.1450 25.6005 0.4435	$x_{14} = x_{21} = x_{32} = x_{43} = 1$; other x_{ij}'s are zero
3	[P(d_2^- /26.4075+ d_3^- /.4743), P(d_1^- /25.3662)]	0.9269 0.8822 0.9999	0.1386490	21.1450 25.6005 0.4435	$x_{14} = x_{21} = x_{32} = x_{43} = 1$; other x_{ij}'s are zero
4	[P(d₁ / 25.3662+ d₂ / 26.4075)), P(d₃ / .4743]	0.9702 1.0000 0.8806	**0.1230585**	20.0455 22.4895 0.5001	$x_{12} = x_{21} = x_{34} = x_{43} = 1$; other x_{ij}'s are zero

9. CONCLUSION

In this paper priority based fuzzy goal programming with generalized trapezoidal fuzzy numbers has been proposed. Euclidean distance is used for selecting proper priority structure for obtaining compromise optimal solution.

The concept presented, in this paper, is illustrated with multi-objective assignment problems involving generalized trapezoidal fuzzy numbers to check the effectiveness of the proposed method. The proposed method is simple and easy to implement. It may be hoped that proposed method can be applied to solve realistic optimization problems involving generalized trapezoidal fuzzy numbers.

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