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λ- Continuous Mappings in Intuitionistic Fuzzy Topological Space

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ABSTRACT

In this paper we introduce intuitionistic fuzzy **λ**-continuous mapping and some of its properties are studied. Also we provide intuitionistic fuzzy **λ**-$T_{1/2}$ space and some of its properties are evolved.

KEYWORDS

Intuitionistic fuzzy topology, intuitionistic fuzzy λ-closed sets, intuitionistic fuzzy λ - open sets, intuitionistic fuzzy λ - continuous mappings and intuitionistic fuzzy **λ**-$T_{1/2}$ space.

AMS SUBJECT CLASSIFICATION (2000).

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1. INTRODUCTION

After the introduction of fuzzy sets by L.A Zadeh [12] in 1965, there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets was introduced by Atanassov [1] in 1986. Using the notion of intuitionistic fuzzy sets, Coker [5] introduced the notion of intuitionistic fuzzy topology in 1997. This approach provides a wide field for investigation in the area of fuzzy topology and its application. The aim of this paper is to introduce the notion of λ -continuous mappings in intuitionistic fuzzy topological space. Moreover we introduced the application of intuitionistic fuzzy λ-closed sets namely, intuitionistic fuzzy **λ**-$T_{1/2}$ space and some of its properties are studied.

2. PRELIMINARIES

Definition 2.1: [1] Let X be a nonempty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{<x, \mu_A(x), \upsilon_B(x) > : x \in X\}$, where the function $\mu_A : X \to [0,1]$ and $\upsilon_A : X \to [0,1]$ denotes the degree of membership $\mu_A(x)$ and the degree of non membership $\gamma_A(x)$ of each element $x \in X$ to the set A respectively and $0 \le \mu_A(x) + \upsilon_A(x) \le 1$ for each $x \in X$.

Definition 2.2[1]: Let A and B be intuitionistic fuzzy sets of the form $\mathbf{A} = \{<x, \mu_A(x), \upsilon_A(x) >: x \in X\}$, and form

$\mathbf{B} = \{<x, \mu_B(x), \upsilon_B(x) >: x \in X\}$.Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x) \rangle / x \in X\}$

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X\}$

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X\}$.

Definition 2.3 [11] : The intuitionistic fuzzy set $c(\alpha, \beta) =< x, c_\alpha, c_{1-\beta} >$ where

$\alpha \in (0,1], \beta \in [0,1)$ and $\alpha + \beta \le 1$ is called an intuitionistic fuzzy point (IFP for short)in X.

Definition 2.4 [11]: Two IFSs are said to be q-coincident ($A_q B$ in short) if and only if there exists an element $x \in X$ such that, $\upsilon_A(x) > \mu_B(x)$ or $\upsilon_A(x) < \mu_B(x)$.

Definition 2.5[5]: An intuitionistic fuzzy topology(IFT for short) on X is a family τ of IFSs in X satisfying the following axioms

(i) $\underset{\sim}{0}, \underset{\sim}{1} \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in I\} \subseteq \tau$

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in τ is known as an intuitionistic fuzzy open set in X.

Definition 2.4 [5]: The complement A^C of an intuitionistic fuzzy open set A in an intuitionistic fuzzy topological space (X, τ) is called intuitionistic fuzzy closed set in X.

Remark 2.5 [5]: For any intuitionistic fuzzy set A in (X, τ), we have

(i) $cl(A^C) = [int(A)]^C$,

(ii) $int(A^C) = [cl(A)]^C$,

(iii) A is an intuitionistic fuzzy closed set in X ⇔ Cl (A) = A

(iv) A is an intuitionistic fuzzy open set in X ⇔ int (A) =A

Definition 2.6[5]: Let (X, τ) be an intuitionistic fuzzy topology and

$A = \{<x, \mu_A(x), \upsilon_B(x) >: x \in X\}$, be an intuitionistic fuzzy set in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

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Int (A) = {G/ G is an intuitionistic fuzzy open set in X and G ⊆ A}

Cl(A) = { K/ K in an intuitionistic fuzzy closed set in X and A ⊆ K }

Definition 2.7 [6]: An intuitionistic fuzzy set A = A = {<x, μ_A (x), υ_B (x) >: x ∈ X} in an intuitionistic fuzzy topology space (X, τ) is said to be

(i) Intuitionistic fuzzy semi closed if int(cl (A) ⊆ A

(ii) Intuitionistic fuzzy pre closed if cl(int(A))⊆ A

Definition 2.9 [9]: An intuitionistic fuzzy set A in an intuitionistic topological space (X, τ) is said to be intuitionistic fuzzy generalized semi-pre closed set (IFGSPCS for short if spcl(A) ⊆ A.

Definition 2.10: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,τ) called

(i). intuitionistic fuzzy generalized closed set [11] (intuitionistic fuzzy g – closed) if cl(A)⊆ U whenever A ⊆ U and U is intuitionistic fuzzy open

(ii) intuitionistic fuzzy g – open set[11], if the complement of an intuitionistic fuzzy g – closed set is called intuitionistic fuzzy g - open set.

(iii) intuitionistic fuzzy semi open (resp. intuitionistic fuzzy semi closed)[6] if there exists an intuitionistic fuzzy open ⊆ (resp. intuitionistic fuzzy closed) suchthat U⊆A ⊆ Cl(U) (resp. int(U) ⊆ A ⊆ U).

Remark 2.11: Every intuitionistic fuzzy closed set [11] (intuitionistic fuzzy open set) is intuitionistic fuzzy g-closed (intuitionistic fuzzy g- open set) but the converse may not be true.

Definition 2.12 [5]: Let X and Y are nonempty sets and f: X→Y is a function.

(a)If B = {< y,, μ_B (y) ,υ_B (y)> : y ∈ Y } is an intuitionistic fuzzy set in Y, then the pre image of B under f denoted by f^{-1}(B) is defined by f^{-1}(B) =={<x,,f^{-1} (μ_B)x ,f^{-1}(υ_B)x : x ∈ X}

(b)If A= {<{x, μ_A (x) ,υ_B (x),)> / x∈ X} is an intuitionistic fuzzy set in X, then the image of A under f denoted by f(A) is the intuitionistic fuzzy set in Y denoted by

f(A) = {<y,,f(μ_A)(y) ,f(υ_A)(y)> : y ∈ Y} where f((υ_A) = 1-f(1-(υ_A)).

Definition 2.13 [6]: Let f :(X, τ) → (Y, σ)

if and if the pre image of each intuitionistic fuzzy open set in Y is an intuitionistic fuzzy topological space Y.

Definition 2.14 [10]: A mapping f:(X , τ)→ (Y, σ) is called an intuitionistic fuzzy generalised semi- pre continuous(IFGSP continuous for short) mapping if f^{-1}(V) is an IFGSPCS in (X , τ).

Through out this paper f :(X, τ) → (Y σ) denotes a mapping from an intuitionistic fuzzy topological space (X, , τ) to another topological space (Y , σ).

Remark 2.15 [11]: Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy g-continuity but the converse may not be true.

Definition 2.16 [8] An intuitionistic fuzzy set A of an intuitionistic topology space (X , τ) is called an

(i) intuitionistic fuzzy λ-closed set (IF λ-CS) if A ⊇ cl(U) whenever A ⊇ U and U is intuitionistic fuzzy open set in X.

(ii) intuitionistic fuzzy λ-open set (IF λ-OS) if the complement A^c of an intuitionistic fuzzy λ-closed set A.

The family of all IF λ-CSs(resp.(IF λ-OSs) of an IFTS (X, τ) is denoted by IF λ-CS(X) (resp.(IF λ-OS(X))

3. INTUITIONISTIC FUZZY λ-CONTINUOUS MAPPINGS

Definition 3.1: A mapping f: (X, τ) → (Y, σ) is said to be intuitionistic fuzzy λ –continuous if the inverse image of every intuitionistic fuzzy closed set of Y is intuitionistic fuzzy λ -closed in X.

Remark 3.2: Every intuitionistic fuzzy continuous is intuitionistic fuzzy λ–continuous but converse may not be true as seen from the following example.

Example 3.3: Let X= {a, b}, Y={x, y} and intuitionistic fuzzy sets U and V are defined aa={<a,0.5,0.5.>,<b,0.6,0.3>}, ={<a,0.5,0.5>, <b,0.2,0.8>}. Let τ -={ $0_\sim$, $1_\sim$,U } and σ ={ $0_\sim$, $1_\sim$,V} be intuitionistic fuzzy topologies on X and Y respectively. Then f: (X, τ) → (Y, σ) defined by f(a)=x and f(b)=y is intuitionistic fuzzy λ–continuity but not fuzzy continuity.

Remark 3.4: The concept of intuitionistic fuzzy λ -continuous mapping and intuitionistic g-continuous mappings are independent as seen from the following examples.

Example3.5: Let X= {a, b}, Y={x, y} and intuitionistic fuzzy sets U and V are defined as follows. U={ <a,0.5,0.5>,<b,0.3,0.6>}, V={<a,0.5,0.5>,<b,0.2,0.6>}. Let τ -= { $0_\sim$,$1_\sim$, U } and σ ={ $0_\sim$ $1_\sim$, V} be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping

f: (X, τ) → (Y, σ) defined by f(a)=x and f(b)=y is intuitionistic fuzzy g-continuity but not intuitionistic fuzzy –λcontinuity

Example 3.6: Let X= {a ,b } and Y={x, y} and intuitionistic fuzzy sets U and V are defined as follows U={<a,0.5,0.5> , <b,0.2, 0.5>} and V={<a,0.5,0.5> , <b,0.4,0.5>}. Let τ = { $0_\sim$ $1_\sim$ U } and σ = { $0_\sim$ $1_\sim$,V } be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping f:(X, τ) → (Y, σ) defined by f(a)=x and f(b)=y is intuitionistic fuzzy λ-continuous but not intuitionistic fuzzy g- continuous.

Remark 3.7: The concept of intuitionistic fuzzy λ-continuous mappings and intuitionistic fuzzy semi continuous

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mappings are independent as seen from the following examples.

Example 3.8: Let X={a ,b,}, Y={x , y} and intuitionistic fuzzy sets U and V are defined as follows: U= {<a, 0.5, 0.5>, <b, 0.2, 0.5 >}, V= {a, 0 .5, 0.5>, <b, 0.4, 0 .5>}.

Let τ ={ $\tilde{0}$ $\tilde{1}$,U } and σ ={ $\tilde{0}$ $\tilde{1}$,V}be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping defined by

f: (X, τ) → (Y, σ) is intuitionisticuzzy -λontinuous but not intuitionistic fuzzy semi continuous.

Example 3.9: Let X={a , b} , Y={x , y} and intuitionistic fuzzy sets U and V are defined as follows: U= {<a, 0. 5, 0 .5 >, <b, 0. 4, 0 .6>} V = {<a, 0.2 .08 >, <b, 0 .1, 0 .9>}.

Let τ = { $\tilde{0}$, $\tilde{1}$,U} and σ = { $\tilde{0}$ $\tilde{1}$,V} } be intuitionistic fuzzy topologies on X and Y respectively then the mapping f: (X, τ) → (Y, σ) defined by f(a)=x and f (b)=y is intuitionistic fuzzy semi continuous mapping but not intuitionistic fuzzy λ - continuous mappings.

Remark 3.10: The concept of intuitionistic fuzzy λ-continuous mappings and intuitionistic fuzzy generalised semi -pre continuous mappings are independent as seen from the following examples.

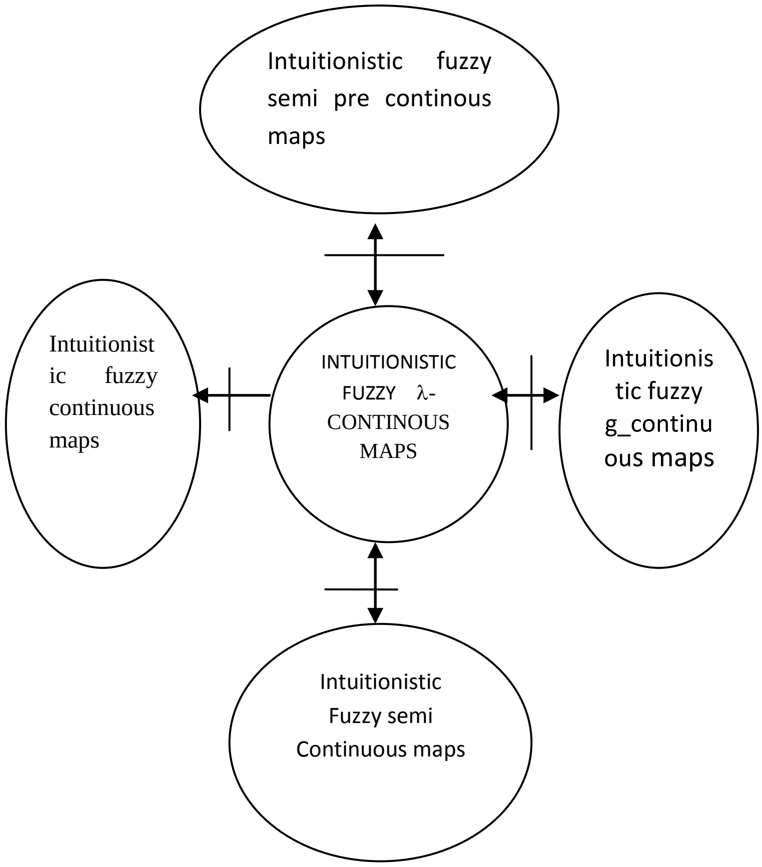
Example 3.11: Let X={a , b} , Y={x , y} and intuitionistic fuzzy sets U and V are defined as follows: U= {<a, 0. 5, 0 .5 >, <b, 0. 5, 0 .3>} V = {<a, 0.5 .0.5 >, <b, 0 .5, 0 .4>}.

Let τ = { $\tilde{0}$, $\tilde{1}$,U} and σ ={ $\tilde{0}$ $\tilde{1}$,V} be intuitionistic fuzzy topologies on X and Y respectively then the mapping f: (X, τ ,) → (Y, σ) defined by f(a)=x and f (b)=y is intuitionistic fuzzy generalized semi -pre continuous mapping but not intuitionistic fuzzy λ- continuous mapping.

Example 3.12: Let X={a , b} , Y={x , y} and intuitionistic fuzzy sets U and V are defined as follows: U= {<a, 0. 5, 0 .5 >, <b, 0. 6, 0 .3>}, V = {<a, 0.5 .0.5 >, <b, 0 .2, 0 .8>}.

Let τ = { $\tilde{0}$, $\tilde{1}$,U} an σ = { $\tilde{0}$ $\tilde{1}$,V } be intuitionistic fuzzy topologies on X and Y respectively then the mapping f: (X, τ) → (Y, σ) defined by f(a)=x and f (b)=y is not intuitionistic fuzzy generalized semi- pre continuous mapping but intuitionistic fuzzy -λ continuous mapping.

Remark 3.13: Remark 3.2, 3.4, 3.9 and 3.10 reveals the following diagram of implication



Theorem 3.14: A mapping f:(X, τ) → (Y , σ) is intuitionistic fuzzy λ-continuous mappings if and only if the inverse image of every intuitionistic fuzzy open set of Y is intuitionistic λ-open set in X

Proof: It is obvious because $f^{-1}(U^c) = [f^{-1}(U)]^c$ for every intuitionistic fuzzy set U of Y.

Theorem 3.15: If f:(X, τ) → (Y , σ) is intuitionistic fuzzy λ- continuous mapping then for each intuitionistic fuzzy point c(α, β) of X and each fuzzy open set V, f (c(α, β) $\subseteq V$ there exist a intuitionistic fuzzy λ -open set U such that c(α, β) ⊆ U and f (U) ⊆ V

Proof: Let c(α, β) be a intuitionistic fuzzy point of X and V be a intuitionistic fuzzy open set such that c(α, β) ⊆ V, put U = f^{-1} (V) then by hypothesis U is intuitionistic fuzzy λ -closed set of X such that c(α, β) ⊆ U and f (U) =f (f^{-1}(V)) ⊆ V.

Theorem 3.16: If f:(X, τ) → (Y , σ) is fuzzy λ-continuous mapping then for each intuitionistic fuzzy point c(α, β) in X and each fuzzy open set V of Y such that $c(\alpha,\beta)_q$) V, there exists c(α, β) in intuitionistic fuzzy λ-open set U of X such that $c(\alpha,\beta)_q$) U and f(U) ⊆ V.

Proof : Let c(α, β) be a intuitionistic fuzzy point of X and V be an intuitionistic fuzzy open set of Y such that, f($c(\alpha,\beta)_q$) V. Put U = f^{-1}(V). Then by hypothesis U is an intuitionistic fuzzy λ-open set of X such that $c(\alpha,\beta)_q$) U and f(U) = f (f^{-1}(V)) ⊆ V.

Remark 3.17: It is clear that A ⊆ cl(A) ⊆ f 1(cl(A)) for any intuitionistic fuzzy set A of X.

Theorem 3.18 : f :(X, τ)→(Y, σ) is intuitionistic fuzzy λ - continuous then

f (cl (A)) ⊆ cl (f(A)) for every intuitionistic fuzzy set A of X.

Proof: Let A be an intuitionistic fuzzy set of X. Then clf(A) is an intuitionistic fuzzy closed set of Y. Since f is intuitionistic fuzzy λ - continuous f^{-1}(cl(A)) is intuitionistic fuzzy λ - closed set in X. Clearly A ⊆ f^{-1} cl(f(A)) Cl(A) ⊆ cl [f^{-1} cl(A))=f^{-1}[clf(A)]. Hence f[cl(A)] ⊆ cl[f(A)].

Theorem 3.19: Let f :(X, τ) →(Y, σ) and g : (Y, σ))→ (Z,γ) be two functions. Then g•f is -λ-continuous if g is continuous and f is λ-continuous.

Proof: Let V be closed set in (Z,γ). Then g^{-1}(V) is closed in (Y, σ). Since g is continuous and f is λ-continuous, f^{-1}(g^{-1}(V)) λ -closed set in (X, τ). But f^{-1}(g^{-1}(V)) = $(g.f)^{-1}$ (V). Then g•f is λ-continuous.

4. APPLICATION OF INTUTIONISTIC FUZZY λ- CONTINUOUS MAPPINGS

De**finition 4.1** A topological space (X, τ) is called intuitionistic fuzzy **λ**-$T_{1/2}$ space (IF **λ**-$T_{1/2}$ **s**pace in short) if every intuitionistic fuzzy λ -closed set is intuitionistic closed in X.

Theorem 4.2: If X is intuitionistic fuzzy **λ**-$T_{1/2}$ space and

f :(X, τ) →(Y, σ) is intuitionistic λ-continuous and then f is continuous.

Proof: Let: f :(X, τ) →(Y, σ) is intuitionistic λ-continuous and let F be any closed set in (Y, σ) Then f (F) is λ -closed set in X. Since f is λ-continuous. But X is IF**λ**-$T_{1/2}$ space. f (F) is closed in X. Hence f is continuous.

Theorem 4.3: If f :(X, τ)→(Y, σ) is intuitionistic fuzzy λ-continuous and g : (Y, σ) → (Z,γ) is an intuitionistic fuzzy continuous mappings and Y is IF**λ**-$T_{1/2}$ -space then g•f : (X, τ) → (Z,γ) is an intuitionistic fuzzy λ-continuous.

Proof: Let V be an intuitionistic fuzzy closed set in Z. Then f^{-1} (V) is an intuitionistic fuzzy closed in Y, by hypothesis. Since f is intuitionistic fuzzy λ-continuous, $f^{-1}(g^{-1}$ (V)) is an intuitionistic fuzzy λ closed in X. But f^{-1}(g^{-1}(V)) = $(g.f)^{-1}$ (V). Hence g•f is an intuitionistic fuzzy λ-continuous.

Theroem :4.4: An IFTS (X, τ) is an IF**λ**-$T_{1/2}$ -space iff IF λ-OS(X) = IFOS(X)

Proof : Let A be an IF λ-open set in X then A^c is an IF λ-closed set in X.By hypothesis

A^c is an IF closed set in X.Therefore A is IF open set in X. Hence IF λ-OS(X) = IF OS(X)

Conversely, let A be IF λ-closed set in X, then A^c is IF λ-open in X.

By assumption A^c is IF open set in X.which in trun implies A is IF closed set in X Hence (X, τ) is an IF**λ**-$T_{1/2}$ -space.

5. CONCLUSION

In this paper we have introduced intuitionistic fuzzy λ-continuous mapping and studied some of its basic properties. Also we have studied the relationship between intuitionistic fuzzy λ-continuos mapping and some of the intuitionistic fuzzy mappings already exist

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