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Basic of p-q Theory for Shunt and Series Compensation

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ABSTRACT
This paper deals with the p-q theory for the generation of reference current in shunt current compensation and reference voltage in series voltage compensation. Basic p-q theory is use to generate reference current and voltage in shunt and series compensation respectively Clarke transformation i.e. α-β-0 transformation and inverse Clarke transformation is use in basic p-q theory. The results are supported by detailed simulation studies on a three phase four-wire compensated system using Matlab.

General Terms
Real power, Imaginary power, Zero sequence power.

Keywords
p-q theory, AC power, Shunt compensation, Series compensation

1. INTRODUCTION
Theory related to instantaneous power is broadly classified in two group first one is define the power on the αβ0 reference frame which is mainly based on the abc to αβ0 formation. Second type defines the power directly in the abc phase. In 1983 Akagi . Konazawa and A. Nabae [1] introduced the instantaneous active and reactive power theory which is also called "p-q theory,". p-q theory is mainly use for the generation of reference current of reactive power compensator, because it define the power clearly. In the p-q theory set of instantaneous powers are define in time domain therefore the behaviors of voltage and current are not restricted. This theory is applicable to three phase system with or without neutral conductor. This theory is valid in both steady state and in transient state. This theory define the power clearly it consider the three phase system together not as a sum of three single phase circuit. p-q theory is commonly used in the power conditioner because it gives the very flexibility designing control strategies and implement in the controller

2. CONCEPT OF POWER IN THREE PHASE CIRCUIT
Due to generation of sinusoidal voltage at constant frequency alternating current (AC) transmission and distribution power system are develops at the end of 19th century. Due to the sinusoidal voltage with constant frequency design of transformer transmission line distribution line are simplified. If the line voltage is not sinusoidal many problem are arrases in design of machines and generated electrification.
When load current is in phase with the source voltage the electric power could be more efficient, due to this the concept of reactive power arrases. Reactive power shows the quantity of electric power due to lagging or leading of load current with source voltage. In the one cycle average reactive power is zero. This means that the reactive power does not take part in energy transfer from source to the load. As soon as concept of reactive power are arrases the concept of apparent power and power factor were created. Apparent power shows the how many power delivered when voltage and current are sinusoidal and both source voltage and current are in phase. Power factor gives the idea about the actual power transfer and the apparent power at the some point. Therefore it is very important to maintain the power factor higher as possible as. Due to high power factor, circuit become more efficient electrically and economically.

3. PHYSICAL MEANIG OF POWER IN THREE PHASE CIRCIT
Physical meaning of the instantaneous real, imaginary and zero sequence powers are explain in figure1

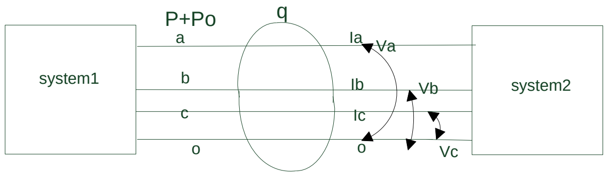

Figure1. Physical meaning of instantaneous power

$p + p_0 =$ Instantaneous total power flow per time unit

q= Total power flow between the phases without transferring power

The all three power has average and oscillating part

$$\text{Real power } p = \bar{p} + \tilde{p}$$
$$\textit{Reactive power } q = \bar{q} + \tilde{q}$$
$$\textit{Zero sequnce power } p_0 = \overline{p_0} - \tilde{p}_o$$

Average and oscillating part of active and reactive part can be expressed as

$$\bar{p} = 3v_+I_+\cos(\emptyset_{v+} - \emptyset_{i+}) + 3v_-I_-\cos(\emptyset_{v-} - \emptyset_{i-})$$

$$\bar{q} = 3v_+I_+\sin(\emptyset_{v+} - \emptyset_{i+}) - 3v_-I_-\cos(\emptyset_{v-} - \emptyset_{i-})$$

$$\tilde{p} = -3v_+I_-\cos(2wt + \emptyset_{v+} - \emptyset_{i-}) - 3v_-I_+\cos(2wt + \emptyset_{v-} - \emptyset_{i+})$$

$$\tilde{q} = -3v_+I_-\sin(2wt + \emptyset_{v+} - \emptyset_{i-}) - 3v_-I_+\sin(2wt + \emptyset_{v-} - \emptyset_{i+})$$

(+) and (-) sign indicate positive and negative sequence respectively .

The zero sequence power can be express as

$$p_0 = \bar{p_o} - \tilde{p}_o$$

$$= 3V_oI_o\cos(\emptyset_{vo} - \emptyset_{io}) - 3V_oI_o\cos(2wt + \emptyset_{vo} + \emptyset_{io})$$

zero sequence power exist only if there are zero sequence voltage and current exist in the zero sequence power there is no way to eliminate the oscillation component because the average zero sequence part is always associated with the oscillating power

From figure1 it is clear that the instantaneous total power flow per unit time is p +p_0. Total power exchanged between the phases without transferring power is reactive or imaginary power q. The real and imaginary power does not affected by zero sequence components. In the fundamental voltage and current total power flow in a unit time is equal to the total real and zero sequence power including both average and oscillating part.

Imaginary power q is not affected by the harmonics and unbalances. It only indicates the power exchange between phases. It is not contribute to the flow of power between the source and load at any time.

4. HISTORY OF p-q THEORY

In 1982 p-q theory firstly publish in the Japanese language in local conference at japan, then it is published in IEEE transaction on industry application 1984 [2]-[5], At the end of 1960 to the beginning of 1970 some paper are publish about the basic idea of compensation of reactive power. In 1976 Gyugyi and Pelly put the theory about compensation of reactive power without energy storage elements using naturally commuted cycloconverter [7][8]. At the same year Gyugyi firstly use the word active AC power filters. In 1980 and 1981 Takahashi A.NABAE and K. Fujiwara gives basic hint to emergence of the p-q theory.

5. THE p-q THEORY

The PQ theory uses Clarke transformation i.e. α-β-0 transformation. Clarke transformation convert three phase voltage and circuit into stationary reference frame by using real matrix are as follows

$$\begin{bmatrix} V_o \\ V_\alpha \\ V_\beta \end{bmatrix} = \sqrt{\frac{3}{2}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_{c\beta} \end{bmatrix}$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & 1 & 0 \\ \frac{1}{\sqrt{2}} & \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{-1}{2} & \frac{-\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} V_o \\ V_\alpha \\ V_\beta \end{bmatrix}$$

The above transformation matrix is also applicable for current transformation

The three phase four wire system is define as follows

$$\begin{bmatrix} p \\ q \\ p_0 \end{bmatrix} = \begin{bmatrix} v_\alpha & v_\beta & 0 \\ -v_\beta & v_\alpha & 0 \\ 0 & 0 & v_o \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix}$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \begin{bmatrix} v_\alpha & v_\beta & 0 \\ -v_\beta & v_\alpha & 0 \\ 0 & 0 & v_o \end{bmatrix}^{-1} \begin{bmatrix} p \\ q \\ p_0 \end{bmatrix}$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \frac{1}{v_0(v_\alpha^2 + v_\beta^2)} \begin{bmatrix} v_\alpha v_o & v_\beta v_o & 0 \\ -v_\beta v_o & v_\alpha v_o & 0 \\ 0 & 0 & (v_\alpha^2 + v_\beta^2) \end{bmatrix} \begin{bmatrix} p \\ q \\ p_0 \end{bmatrix}$$

$$\begin{bmatrix} i_{\alpha p} \\ i_{\beta p} \\ i_{0p} \end{bmatrix} + \begin{bmatrix} i_{\alpha q} \\ i_{\beta q} \\ i_{0q} \end{bmatrix} + \begin{bmatrix} i_{\alpha p_o} \\ i_{\beta p_o} \\ i_{0p_o} \end{bmatrix} = \frac{\begin{bmatrix} v_\alpha & v_\beta & 0 \\ -v_\beta & v_\alpha & 0 \\ 0 & 0 & \frac{(v_\alpha^2 + v_\beta^2)}{v_0} \end{bmatrix}}{(v_\alpha^2 + v_\beta^2)} \begin{bmatrix} p \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ q \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ p_0 \end{bmatrix}$$

$$i_{\alpha p} = \alpha - axis\ instantaneous\ active\ current = \frac{v_{\alpha p}}{v_\alpha^2 + v_\beta^2} p$$

$$i_{\alpha q} = \alpha - axis\ instantaneous\ reactive\ current = \frac{v_{\alpha q} q}{v_\alpha^2 + v_\beta^2}$$

$$i_{\alpha 0} = \alpha - \text{axis instantaneous zero sequnce current} = 0$$

$$i_{\beta p} = \beta - axis\ instantaneous\ active\ current = \frac{-v_{\beta p}}{v_\alpha^2 + v_\beta^2} p$$

$$i_{\beta q} = \beta - axis\ instantaneous\ reactive\ current = \frac{v_{\beta q} q}{v_\alpha^2 + v_\beta^2}$$

$$i_{\beta q} - \beta - \text{axis instantaneous zero sequnce current} = 0$$

$$i_{0p} = 0 - axis\ instantaneus\ active\ current = 0$$

$$i_{0q} = 0 - axis\ instantaneus\ reactive\ current = 0$$

$$i_{0p_o} = 0 - axis\ zero\ sequnce\ current = \frac{p_0}{v_0}$$

by using the equation of above instantaneous reactive current instantaneous power are express follows

$$p_{\alpha p} = \alpha - axis\ instantaneous\ active\ power = v_\alpha i_{\alpha p}$$

$$= \frac{v_\alpha^2}{v_\alpha^2 + v_\beta^2} p$$

$$p_{\alpha q} = \alpha - axis\ instantaneous\ reactive\ power = v_\alpha i_{\alpha q}$$

$$= \frac{v_\alpha v_\beta}{v_\alpha^2 + v_\beta^2} q$$

$$p_{\alpha 0} = \alpha - axis\ instantaneus\ zerosequnce\ power = 0$$

$$p_{\beta p} = \beta - axis\ instantaneous\ active\ power = v_\beta i_{\beta p}$$

$$= \frac{v_\beta^2}{v_\alpha^2 + v_\beta^2} p$$

$$p_{\beta q} = \beta - \text{axis instantaneous reactive power} = v_\beta i_{\beta q}$$

$$= \frac{v_\alpha v_\beta}{v_\alpha^2 + v_\beta^2} q$$

$$p_{\beta 0} - \beta - \text{axis instantaneous zero sea power} = 0$$

$$p_{0p} - 0 - \text{axis instantaneus active power} = 0$$

$$p_{0q} - 0 - \text{axis instantaneus reactive power} = 0$$

$$p_{0p_0} - 0 - \text{axisin stantaneus reactive power} = v_0 i_{0p_0}$$

6 APPLICATION OF pq THEORY

6.1 SHUNT CURRENT COMPENSATION BY p-q THEORY

In the shunt current compensation Basic concept is compensation of unwanted current that is compensator inject the current such that it cancel the unwanted current in system

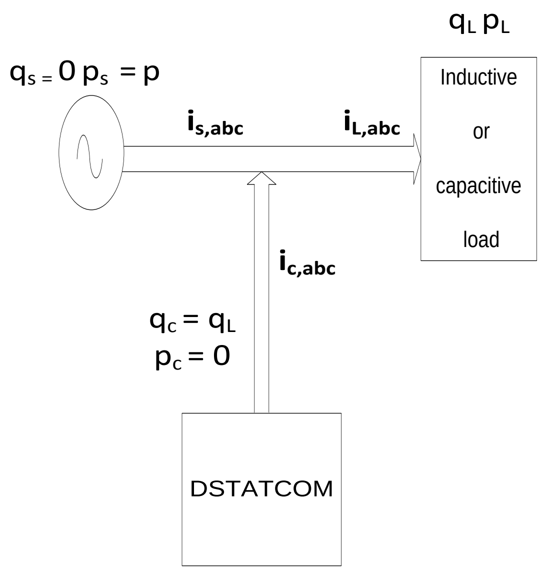

Figure2. Simplest form of shunt compensator

Figure2 show the simplest form of shunt compensator $I_{s,abc}$,show the source current . $I_{l,qbc,}$ Show the load current and $I_{c,abc}$ show the compensator current. Shunt compensator behaves as three-phase controller current source. It generates compensator current as per requirement of system.

Figure 3 Show the basic control scheme used in controller of shunt compensator. In this firstly calculate the real and reactive power then both divided into overage (φ) and oscillating part from these powers calculate the reference current.

There is no any exchange of real power in compensator there for compensator does not required any power source or energy storage system.

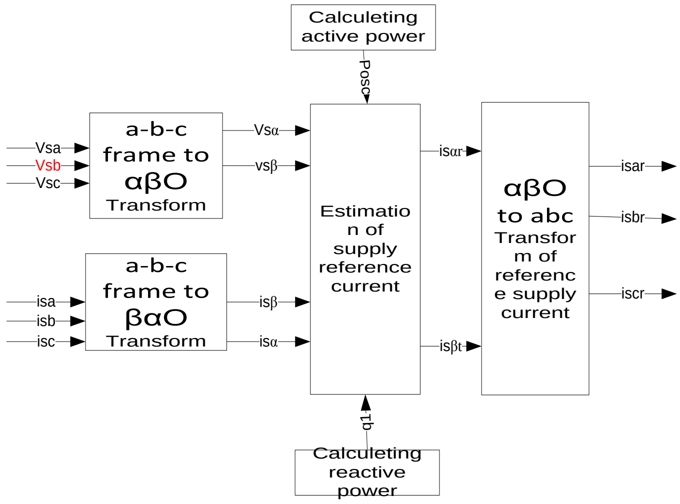

Figure3. The basic scheme of reference current generation in shunt compensator

6.2 SERIES VOLTAGE COMPENSATION USING p-q THEORY

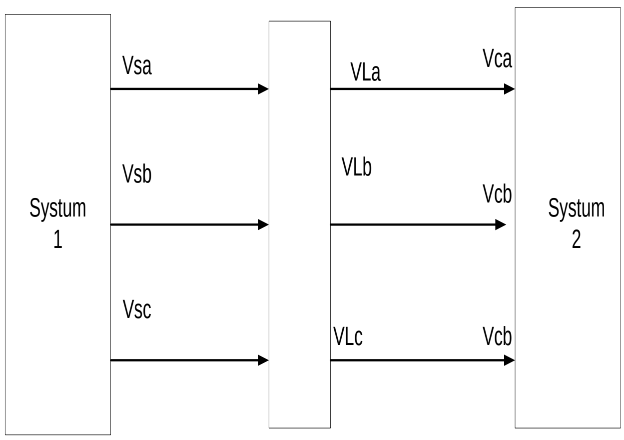

Figure 4 series voltage compensation.

Figure 4 show the basic configuration of series voltage compensation. Where Vsa, Vsb, Vsc, are the source voltage Vca, Vcb, Vcc, are the compensator voltage and V_{la} , V_{lb} , V_{lc} , are the load voltage . Series compensator behaves as control voltage source

In shunt current compensation current component calculate as function of the voltage, active and reactive power but in the series voltage compensation voltage component calculate as a function of current and reactive power.

The active power, reactive power and zero sequence power are define from the instantaneous voltage and current on the α-β-0axes as

$$\begin{bmatrix} p \\ q \\ p_0 \end{bmatrix} = \begin{bmatrix} i_\alpha & i_\beta & 0 \\ -i_\beta & i_\alpha & 0 \\ 0 & 0 & i_o \end{bmatrix} \begin{bmatrix} v_\alpha \\ v_\beta \\ v_0 \end{bmatrix}$$

$$\begin{bmatrix} v_\alpha \\ v_\beta \\ v_0 \end{bmatrix} = \begin{bmatrix} i_\alpha & i_\beta & 0 \\ -i_\beta & i_\alpha & 0 \\ 0 & 0 & i_o \end{bmatrix}^{-1} \begin{bmatrix} p \\ q \\ p_0 \end{bmatrix}$$

$$\begin{bmatrix} v_\alpha \\ v_\beta \\ v_0 \end{bmatrix} = \frac{1}{i_0({i_\alpha}^2+{i_\beta}^2)} \begin{bmatrix} i_\alpha i_o & i_\beta i_o & 0 \\ -i_\beta i_o & i_\alpha i_o & 0 \\ 0 & 0 & ({i_\alpha}^2+{i_\beta}^2) \end{bmatrix} \begin{bmatrix} p \\ q \\ p_0 \end{bmatrix}$$

$$\begin{bmatrix} v_\alpha \\ v_\beta \\ v_0 \end{bmatrix} = \frac{\begin{bmatrix} i_\alpha i_o & i_\beta i_o & 0 \\ -i_\beta i_o & i_\alpha i_o & 0 \\ 0 & 0 & \frac{({i_\alpha}^2+{i_\beta}^2)}{i_0} \end{bmatrix}}{({i_\alpha}^2+{i_\beta}^2)} \begin{bmatrix} p \\ q \\ p_0 \end{bmatrix}$$

$$\begin{bmatrix} v_{\alpha p} \\ v_{\beta p} \\ v_{0p} \end{bmatrix} + \begin{bmatrix} v_{\alpha q} \\ v_{\beta q} \\ v_{0q} \end{bmatrix} + \begin{bmatrix} v_{\alpha p_0} \\ v_{\beta p_0} \\ v_{0 p_0} \end{bmatrix} =$$

$$\frac{1}{({i_\alpha}^2+{i_\beta}^2)} \begin{bmatrix} i_\alpha & i_\beta & 0 \\ -i_\beta & i_\alpha & 0 \\ 0 & 0 & \frac{({i_\alpha}^2+{i_\beta}^2)}{i_0} \end{bmatrix} \begin{bmatrix} p \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ q \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ p_0 \end{bmatrix}$$

$$v_{\alpha p} = \alpha - axis\ instantaneous\ active\ voltage = \frac{i_{\alpha p}}{{i_\alpha}^2 + {i_\beta}^2} p$$

$$v_{\alpha p} = \alpha - axis\ instantaneous\ reactive\ voltage = \frac{i_{\alpha p}}{{i_\alpha}^2 + {i_\beta}^2} q$$

$$v_{\alpha 0} = \alpha - \text{axis instantaneous zero sequnce voltage} = 0$$

$$v_{\beta p} = \beta - axis\ instantaneous\ active\ voltage = \frac{i_{\beta p}}{{i_\alpha}^2 + {i_\beta}^2} p$$

$$v_{\beta q} = \beta - axis\ instantaneous\ active\ voltage = \frac{i_{\beta q}}{{i_\alpha}^2 + {i_\beta}^2} q$$

$$v_{\beta 0} = \beta - \text{axis instantaneous zero sequnce voltage} = 0$$

$$v_{0p} = 0 - \text{axis instantaneous active voltage} = 0$$

$$v_{0q} = 0 - \text{axis instantaneous reactive voltage} = 0$$

$$v_{0p_o} = 0 - \text{axis instantaneous zero sequnce voltage} = \frac{p_0}{i_0}$$

7. Control sachem for series voltage compensation

Figure 5 Show the control sachem for series voltage compensation base on the pq theory the compensating reference voltage are directly calculate from load terminal and connected from abc to α-β-0 frame and calculate the active and reactive power and estimate the reference current, then convert these current from α-β-0 frame to abc frame.

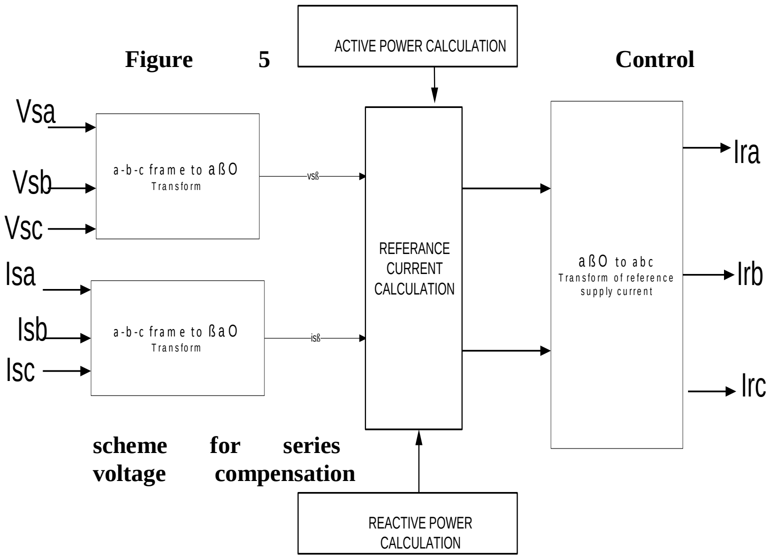

Figure 5 Control scheme for series voltage compensation

8. SIMULATION RESULT

In order to analyze the performance of this control scheme, we done Matlab simulation with following parameter

Vs= 380, f=50Hz, Rs=1Ω, Ls= 5mH, Load = 11 KW

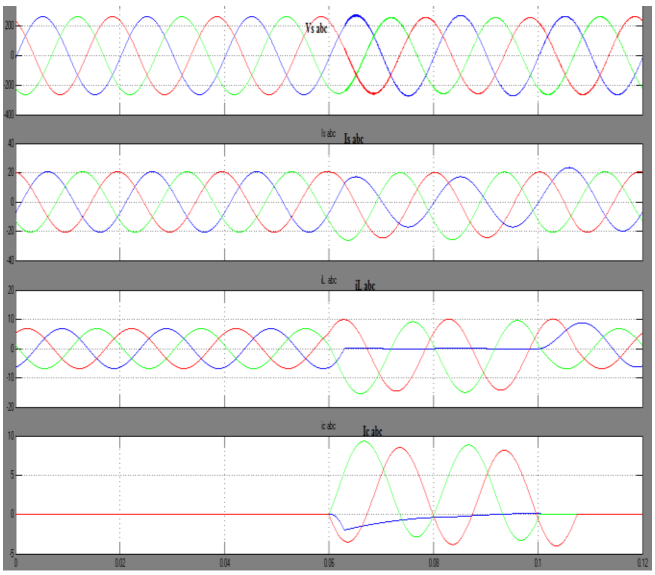

Figure 6. Simulation Results for COMPENSTOR

Figure 6 shows the simulation of compensator in which pq theory use for reference current generation. Initially at normal condition compensator inject balance current and as soon as load current become unbalance compensator inject current as per requirement.

CONCLUSION

In this paper a basic p-q theory using Clarke transformation has been presented. In this theory reference currents and voltages in shunt and series compensation respectively are generated on the basis of average and oscillating component of power.

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