International Journal of Applied Information Systems (IJAIS) – ISSN : 2249-0868
Foundation of Computer Science FCS, New York, USA
International Conference and Workshop on Communication, Computing and Virtualization (ICWCCV 2015) – www.ijais.org

Hybrid Synchronization of Fractional Order Hyper Chaotic Chen and Lu Systems using Active Control Method

Vijay K. Yadav
Department of Mathematical Sciences,
Indian Institute of Technology (BHU), Varanasi-221005India

Prashant K. Mishra
Department of Mathematical Sciences,
Indian Institute of Technology (BHU), Varanasi-221005India

Vivek Mishra
Department of Mathematical Sciences,
Indian Institute of Technology (BHU), Varanasi-221005India

Mayank Srivastava
Department of Mathematics and Statistics,
SRSM University, Lucknow, India.

Saurabh K. Agrawal
Department of Applied Sciences, Bharati Vidyapeeth's College of Engineering, Delhi-110063, India

ABSTRACT
In this article, the active control method is used for hybrid synchronization between two fractional order hyperchaotic systems, Chen hyperchaotic system taken as the master system and the other fractional order hyperchaoticLu system taken as slave system separately. The fractional derivative is described in Caputo sense. Numerical simulation results which are carried out using Adams–Bashforth–Moulton method show that the method is easy to implement and reliable for hybrid synchronizing the two nonlinear fractional order chaotic systems while it also allows both the systems to remain in chaotic states.

Key word
Fractional order, Hyperchaotic system, Active control method, Synchronization, Anti-synchronization.

1. INTRODUCTION
The concept of dynamical system originates in the Newtonian mechanics .The mathematical model of the dynamical system are generally represented in the form of differential equation.

In start modelling was limited to linear dynamical system but slowly it extended to the nonlinear dynamical system,

since nonlinearity is aimportant feature of the real life situation.
Due to advancement in the computational machines it possible now to touch the study of nonlinear dynamical systems. Chaos in nonlinear dynamical system is a interesting topic of research now days. With the advancement of fractional calculus, fractional order dynamical system is a new avenue for the researchers. Fractional calculus has its age as old as calculus around 300 years old. It is believed that the origin of fractional calculus happened in a later conversation between Leibniz and L'Hospital. Fractional order operator can deal with the system where system correlates with the long range of time. The two major advancement in the field of dynamical systems are predator and prey model and Lotkavolteramodel. Predator Prey model are used in the field of Ecology where asLotkaVoltera model are used in the population dynamics. Synchronization of two dynamical system represents the situation when one dynamical system mimics another dynamical system.Pecora and Carroll [1], first introduced a method about synchronization between the drive (master) and

response (slave) systems for two identical or non-identical systems with different initial conditions.In last few years varioussynchronization schemes, such as linear and nonlinearfeedback synchronization [2], time delay feedback approach[3], adaptive control [4], active control[5], etc. have been applied for synchronizationofchaoticsystem.Theauthors' have made an effort to study thehybrid synchronization for two different types fractional order systems. We started off considering the fractional order hyperchaotic Chen system as drive system and the fractional order hyperchaotic Lu as response system. It is best to the author's knowledge that hybrid synchronization of these two systems by the active control method is not yet done by any researcher.

2. SYSTEMS' DESCRIPTION
2.1 Fractional Order Hyper Chaotic Chen System
The fractional order hyperchaotic Chen system [7, 8] is given by

$$\frac{d^q x_1}{dt^q} = a_1(y_1 - x_1) + w_1$$
$$\frac{d^q y_1}{dt^q} = d_1 x_1 - x_1 z_1 + c_1 y_1$$
$$\frac{d^q z_1}{dt^q} = -x_1 y_1 - b_1 z_1 \quad (1)$$
$$\frac{d^q w_1}{dt^q} = -y_1 z_1 + r_1 w_1,$$

where x_1, y_1, z_1 and w_1 are states variables and a_1, b_1, c_1, d_1 and r_1 are the constant parameters. At the value of parameters $a_1 = 35, b_1 = 3, c_1 = 28, d_1 = 7$ and $r_1 = 0.5$ the Chen system shows hyperchaoticbehaviour. The phase portraits of (1) in $x_1 - y_1 - z_1$, $x_1 - y_1 - w_1$, $x_1 - z_1 - w_1$

and $y_1 - z_1 - w_1$ space are depicted through Fig.1 for $q = 0.97$

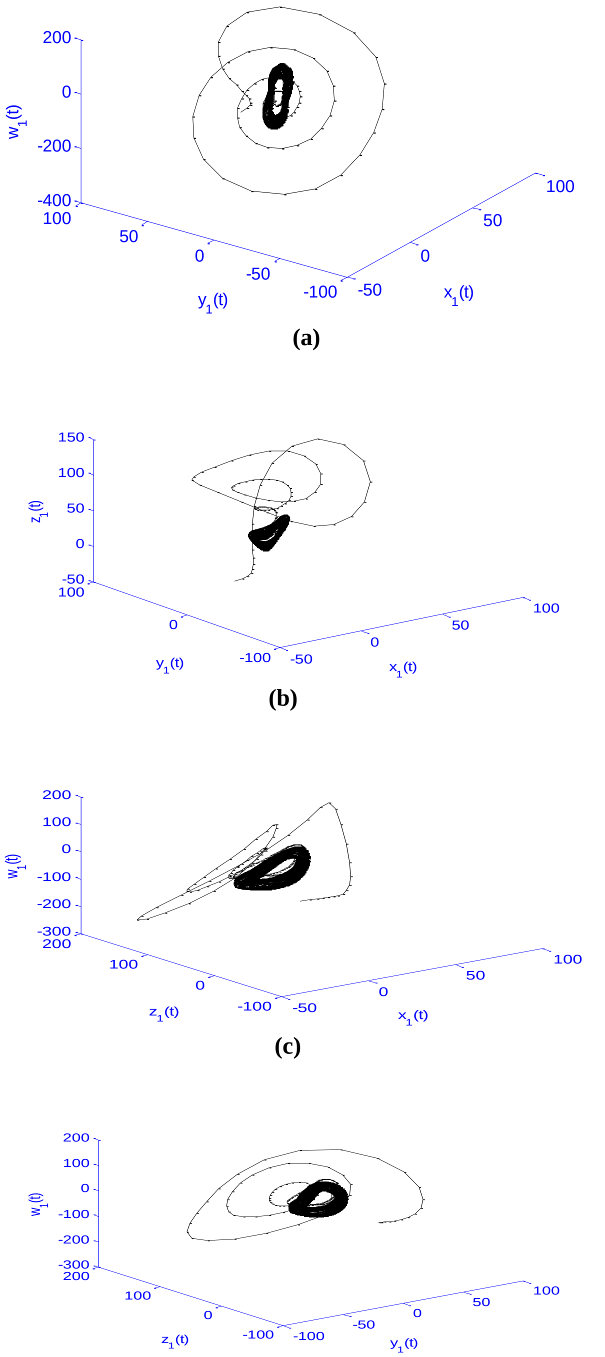


Fig. 1. Phase portraits of hyperchaotic Chen system for $q = 0.97$ **(a) in** $x_1 - y_1 - z_1$ **space (b) in** $x_1 - y_1 - w_1$ **space (c) in** $x_1 - z_1 - w_1$ **space (d) in** $y_1 - z_1 - w_1$ **space**

2.2 Fractional Order Hyper chaotic Lu System

The fractional order hyperchaotic Lu [7, 9] system is given as

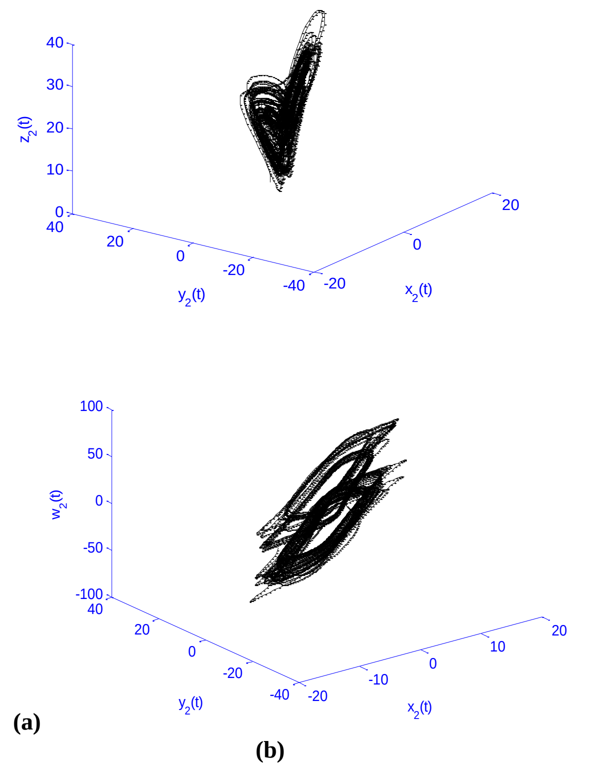
$$\frac{d^q x_2}{dt^q} = a_2(y_2 - x_2) + w_2$$

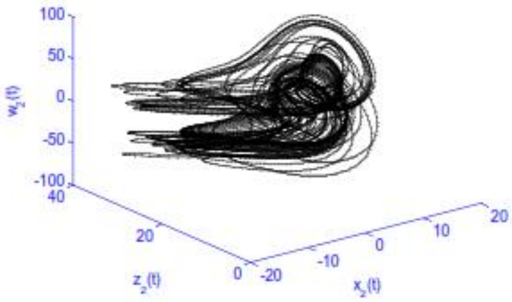
$$\frac{d^q y_2}{dt^q} = -x_2 z_2 + c_2 y_2$$

$$\frac{d^q z_2}{dt^q} = x_2 y_2 - b_2 z_2 \qquad (2)$$

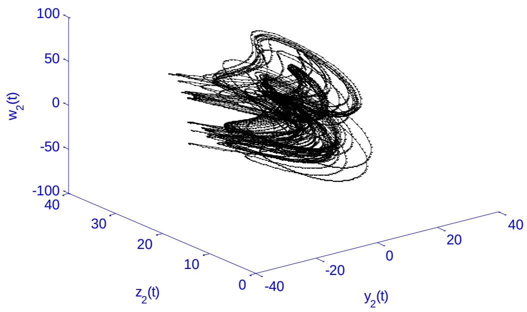
$$\frac{d^q w_2}{dt^q} = x_2 z_2 + d_2 w_2,$$

where x_2, y_2, z_2 and w_2 are states variables and a_2, b_2, c_2 and d_2 are parameters. When $a_2 = 36, b_2 = 3, c_2 = 20$ and $-0.35 \le d_2 \le 130$, the Lu system has a chaotic attractor. Phase portraits of fractional order hyperchaotic Lu system are shown in Fig. 2 at $q = 0.97$





(c)



(d)

Fig. 2. Phase portraits of hyperchaotic Lu system for $q = 0.97$ (a) in $x_2 - y_2 - z_2$ space (b) in $x_2 - y_2 - w_2$ space (c) in $x_2 - z_2 - w_2$ space (d) in $y_2 - z_2 - w_2$ space

3. HYBRID SYNCHRONIZATION BETWEEN FRACTIONAL ORDER HYPERCHAOTIC CHEN AND LU SYSTEMS

In this section we study the hybrid synchronization between two fractional order hyperchaoticChen and Lu Systems using active control method. Here we take fractional order hyperchaotic Chen system as a master system as

$$\frac{d^q x_1}{dt^q} = a_1(y_1 - x_1) + w_1$$

$$\frac{d^q y_1}{dt^q} = d_1 x_1 - x_1 z_1 + c_1 y_1$$

$$\frac{d^q z_1}{dt^q} = -x_1 y_1 - b_1 z_1 \quad (3)$$

$$\frac{d^q w_1}{dt^q} = -y_1 z_1 + r_1 w_1,$$

and fractional order hyperchaotic Lu system taken as slave system

$$\frac{d^q x_2}{dt^q} = a_2(y_2 - x_2) + w_2 + u_1(t)$$

$$\frac{d^q y_2}{dt^q} = -x_2 z_2 + c_2 y_2 + u_2(t) \quad (4)$$

$$\frac{d^q z_2}{dt^q} = x_2 y_2 - b_2 z_2 + u_3(t)$$

$$\frac{d^q w_2}{dt^q} = x_2 z_2 + d_2 w_2 + u_4(t),$$

where $u_1(t)$, $u_2(t)$, $u_3(t)$ and $u_4(t)$ are control function. The error states defined as

$e_1 = x_2 - x_1$, , $e_3 = z_2 - z_1$ and $e_4 = w_2 + w_1$, then we obtain the following error systems from equations (3) and (4) according to defining errors as

$$\frac{d^q e_1}{dt^q} = a_2(e_2 - e_1) + e_4 - (a_2 - a_1)x_1 - (a_2 + a_1)y_1 - 2w_1 + u_1(t)$$

$$\frac{d^q e_2}{dt^q} = c_2 e_2 + d_1 x_1 + (c_1 - c_2)y_1 - x_2 z_2 - x_1 z_1 + u_2(t)$$

$$\frac{d^q e_3}{dt^q} = -b_2 e_3 + (b_1 - b_2)z_1 + x_2 y_2 - x_1 y_1 + u_3(t)$$

$$\frac{d^q e_4}{dt^q} = d_2 e_4 - (d_2 - r_1)w_1 + x_2 z_2 + y_1 z_1 + u_4(t).$$

Now we define the active control function $u_1(t)$, $u_2(t)$, $u_3(t)$ and $u_4(t)$ as

$$u_1(t) = (a_2 - a_1)x_1 + (a_2 + a_1)y_1 + 2w_1 + v_1(t)$$

$$u_2(t) = -d_1 x_1 - (c_1 - c_2)y_1 + x_2 z_2 + x_1 z_1 + v_2(t)$$

$$u_3(t) = -(b_1 - b_2)z_1 - x_2 y_2 + x_1 y_1 + v_3(t)$$

$$u_4(t) = (d_2 - r_1)w_1 - x_2 z_2 - y_1 z_1 + v_4(t)$$

which leads to errors function as

$$\frac{d^q e_1}{dt^q} = a_2(e_2 - e_1) + e_4 + v_1(t)$$

$$\frac{d^q e_2}{dt^q} = c_2 e_2 + v_2(t)$$

$$\frac{d^q e_3}{dt^q} = -b_2 e_3 + v_3(t) \quad (5)$$

$$\frac{d^q e_4}{dt^q} = d_2 e_4 + v_4(t),$$

where the $v_1(t)$, $v_2(t)$, $v_3(t)$ and $v_4(t)$ are active controls chosen such way that the system (5) become stable. We choose

$$\begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \\ v_4(t) \end{bmatrix} = A \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix},$$

where A is a 4×4 constant matrix. In order to make the closed loop system stable,matrixAshould be selected in such a way that the feedback system has eigenvalues λ_i of A which satisfy the condition $|\arg(\lambda_i)| > (q\pi/2),\ i = 1,2,3,4$. There is no unique choice formatrix*A*, but a good choice can be as follows

$$A = \begin{bmatrix} a_2 - 1 & -a_2 & 0 & -1 \\ 0 & -c_2 - 1 & 0 & 0 \\ 0 & 0 & b_2 - 1 & 0 \\ 0 & 0 & 0 & -d_2 - 1 \end{bmatrix},$$

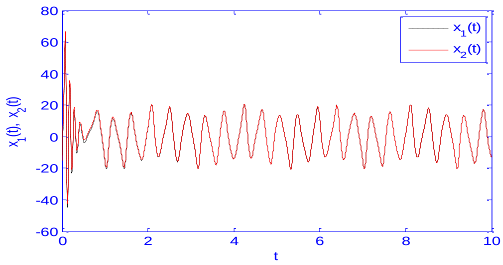
Thus the errors system (5) is changed to

$$\frac{d^q e_1}{dt^q} = -e_1$$

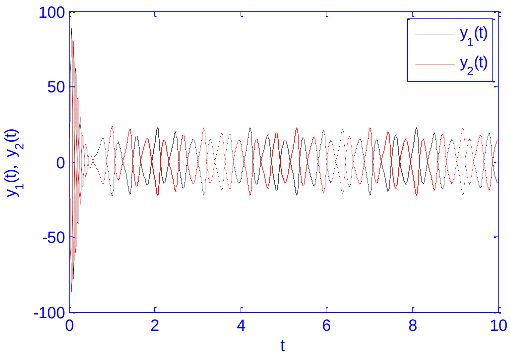
$$\frac{d^q e_2}{dt^q} = -e_2$$

$$\frac{d^q e_3}{dt^q} = -e_3 \qquad (6)\ \frac{d^q e_4}{dt^q} = -e_4 .$$

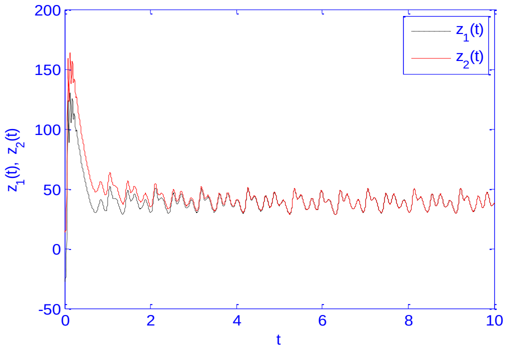
All the eigenvalues of the error systems (6) are -1 and hence the condition $|\arg(\lambda_i)| > (q\pi/2)$, for $0 < q \le 1$ is satisfied. Therefore the systems are stable and required hybrid synchronization is obtained.



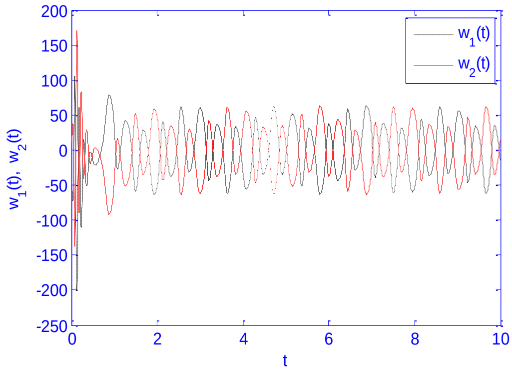
(a)



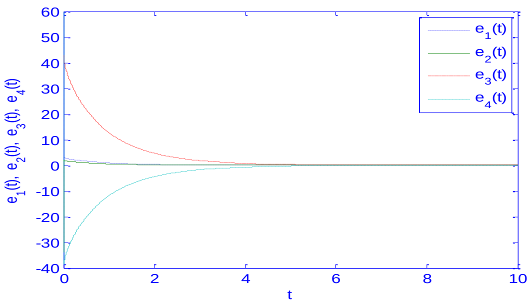
(b)



(c)



(d)



(e)

Fig.3. Hybrid **S**ynchronization for signals (a) between x_1 and x_2 (b) between y_1 and y_2 (c) between z_1 and z_2 (d) w_1 and w_2 (e)The evolution of the error functions of hyperchaotic systems, for fractional order derivative $q = 0.97$

4. NUMERICAL SIMULATION

In the numerical simulation, the parameters of the fractional order hyperchaotic Chen system are taken as $a_1 = 35,\ b_1 = 3, c_1 = 28,\ d_1 = 7$ and $r_1 = 0.5$. Parameters of fractional order hyperchaotic Lu system are taken as $a_2 = 36,\ b_2 = 3, c_2 = 20$ and $d_2 = -0.35$, respectively. The initial conditions of the master systems are taken as $(-12,\ 16,\ -28,\ -50)$ and slave system is taken as $(-15,\ -18,\ 15,\ 10)$ respectively. Hence the initial conditions of error system will be $(-3,\ -34,\ 43,\ 60)$. The time step size is taken as 0.005.

State trajectory of hybrid synchronization master and slave systems are depicted through Fig. 3 at order $q = 0.97$

5. CONCLUSION

In present articlethe major contribution of the authors is achieved by hybrid synchronization between hyperchaotic Chen and Lu systems in fractional order through active control method. In this method the control functions are designed through error states tend to zero when time become large.For numerical simulation results are carried out using Adams BashforthmoultenMethod with help of MATLAB.

6. REFERENCES

[1] Pecora, L.M. and Carroll, T.L.1990. Synchronization in chaotic systems. PhysRevLett. 64:821–4.

[2] Chen, S.H. and Lu J. 2002.Parameters identification and synchronization ofchaotic systems based upon adaptive control. PhysLett A, 299:353–8

[3] Park, J.H. and Kwon, O.M. 2005.A novel criterion for delayed feedback control of time-delay chaotic systems. Chaos SolutionFract, 23: 495-01.

[4] Chen, S.Hand Lu, J. 2002.Synchronization of an uncertain unified chaotic system via adaptive control. Chaos SolitonFract, 14:643–7.

[5] Ho, M.C. and Hung, Y.C. 2002.ynchronization of two different systems by using generalized active control. PhysLett A, 301:424–9.

[6] I.Podlubny.1999. Fractional differential equations. San Diego, New York: Academic Press.

[7] Lin Pan, Wuneng Zhou, Long Zhou, and Kehui Sun.2011. Chaos synchronization between two different fractional-order hyperchaotic systems, Commun Nonlinear SciNumerSimulat, 16:2628–2640.

[8] Sudheer, S. K. and Sabir, M.2009. Hybrid Synchronization of Hyperchaotic Chen System,

[9] National Conference on Nonlinear Systems & Dynamics, Saha Institute of Nuclear Physics, Kolkata, India.

[10] Agrawal, S. K. and Das, S. 2013Projective synchronization between different fractional-order hyperchaotic systems with uncertain parameters using proposed modified adaptive projective synchronization technique, Mathematical Methods in Applied Sciences, DOI: 10.1002/mma.2963.

[11] SudheerK. S. and Sabir, M. 2009. Hybrid synchronization of hyperchaotic Lu system, PRAMANA journal Physics, 73, (October 2009), 781-786.

[12] Agrawal, S. K., Srivastava, M. and Das,S.2012.Synchronization of fractional order chaotic systems using active control method, Chaos, Solitons& Fractals, 45:737–752.