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A Novel Segmentation Approach by the Concept of Stability Analysis

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Abstract
The most crucial step in the process of image processing is image segmentation. Though different image segmentation algorithms have been developed, yet each method has its own advantages and limitations. In this paper a novel approach for image segmentation has been proposed based on the concept of stability. The concept of stability in terms of modified Sylvester's formula applied in the context of positive definiteness, semi positive definiteness and negative definiteness to satisfy a region of convergence is applied for image segmentation. The proposed methodology is applied for images with distinct homogeneous regions. The segmentation precision is quantified and it is evident through visual inspection.

Keywords
Segmentation, Stability, Positive definite, Semi definite, Negative definite, Region of Convergence

I. INTRODUCTION
The purpose of image segmentation is to partition an image into meaningful regions with respect to a particular application. This stage of image processing is an important step which divides an image into regions with similar properties such as gray level, color, texture, brightness and contrast. If the image contains only homogeneous color regions, clustering methods in color space are sufficient to handle the problem. In reality, natural scenes are rich in color and texture. In color texture patterns, it is difficult to identify image regions. The different approaches in image segmentation can be classified in [1] as Contour Based methods, like active contours and active shape models; Region based techniques; Watershed Transform; Optimization methods; Clustering methods, like k-means, Fuzzy C-means, Hierarchical clustering; Thresholding methods.

The active contour models has two different paths in terms of representation and implementation namely parametric active contours (PACs) which uses parametric representation of curves and geometric active contours (GACs) which utilize level set methods [2,3]. The topological changes of the evolving contour such as splitting and merging andsingularities on the curve such as sharp corners are handled in level set methods [4]. The problem of boundary extraction, motion tracking and segmentation are successfully implemented through level set method [5],[6], [7]. The results obtained is sensitive to initialization conditions and the segmentation techniques based on topological derivative needs a post processing step in order to improve the results.

Watershed transform is a useful segmentation technique because of its simplicity, parallelization and providing complete division of image accompanied by the drawback of over segmentation. The K-means is a popular clustering algorithm to find the clustering by iteration [10], [11]. But the computational complexity of the traditional k-means due to accessing the whole data in each cycle of iterative operations is too great to make it fit for very large data set. Another challenge is the incorporation of strong assumptions (often multivariate Gaussian) about the multidimensional shape of clusters to be obtained. The clustering techniques include relevance feedback [12], log based clustering [13], hierarchical clustering [14], graph based, retrieval-dictionary based, filter based clustering etc.

A novel approach is thus proposed based on the concept of stability. The Sylvester's theorem for positive definitefunctions [8],[9] is slightly modified and applied to find the region which is not convergent with the region of convergence to detect the unstable region which lies outside the region of convergence. This method of segmentation is region based and hence does not allow over segmentation or erroneous results due to noisy pixels.

The paper is organized as follows. In section II the Modified Sylvester's theorem for positive function is explained and the modification proposed for the problem of image segmentation is explained. The concept of positive, semi positive and negative definite functions is explained with its applications to image segmentation is proposed in section III. Section IV includes results and discussions and section V gives the conclusion and further work.

II. MODIFIED SYLVESTER'S METHOD
Sylvester's method is used for testing whether the given function is positive definite or not. Let the function be expressed as a polynomial

$$V(x) = a_{11}x_1^2 + 2a_{12}x_1x_2 + a_{22}x_2^2 + 2a_{13}x_1x_3 + a_{33}x_3^2 + 2a_{23}x_2x_3 \quad (1)$$

where a_{11}, a_{12}, a_{22}, a_{13}, a_{33}, a_{23} are the coefficients of the equation.
This function can be expressed as a matrix

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad (2)$$

Here $a_{21}=a_{12}$, $a_{13}=a_{31}$, $a_{23}=a_{32}$
For a function to be positive definite,

$$\left|a_{11}\right| > 0$$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} > 0 \quad \Rightarrow a_{11}a_{22} - a_{12}a_{21} > 0$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} > 0 \Rightarrow a_{11}\left(a_{22}a_{33} - a_{23}a_{32}\right) - a_{12}\left(a_{21}a_{33} - a_{31}a_{23}\right) + a_{13}\left(a_{21}a_{32} - a_{22}a_{31}\right) > 0 \tag{3}$$

$$\left|a_{11}\right| > 0$$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} > 0 \quad \Rightarrow a_{11}a_{22} + a_{12}a_{21} > 0$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} > 0 \Rightarrow a_{11}\left(a_{22}a_{33} + a_{23}a_{32}\right) - a_{12}\left(a_{21}a_{33} + a_{31}a_{23}\right) + a_{13}\left(a_{21}a_{32} + a_{22}a_{31}\right) > 0 \tag{4}$$

If all the three conditions of (3) are satisfied, then the given function is positive definite. If any of the three conditions are not positive then the given function is negative definite. This concept of Sylvester's theorem is applied in image segmentation to find asymptotically stability. But the problem with the application of original Sylvester's theorem is that the above three conditions for positive definiteness of a homogeneous, gray, black and white regions will result in erroneous conclusion. The result obtained for the second and third conditions will be the same for all homogeneous, gray, black and white regions. Hence for the application to segmentation problem, the Sylvester's theorem has been modified slightly as in (4) to incorporate the changes in the different homogeneous images.

This slight modification in Sylvester's theorem has eliminated the problem of misinterpretation.

III. PROPOSED METHOD

The problem of image segmentation is the process of outlining a relevant data set in the image. To segment a region in the image a seed pixel in the region is chosen. A region of convergence (ROC) is formed around the seed pixel by forming a matrix.

The region of convergence is

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{0} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} \tag{5}$$

Where,

S_0 – seed pixel

S_{11},. ……S_{33} – pixels adjacent to the seed pixel

This ROC is taken as the seed region to be segmented. The segmentation process is started by forming regions of adjacency with region of convergence. The regions of adjacency are formed by forming matrices on the horizontal and vertical directions (both right, left and up, down).

The regions of adjacency are formed and the modified Sylvester's theorem for positive definiteness is applied to test whether the centre pixel of each adjacency matrix is asymptotically stable or marginally stable or unstable.

Consider the region of adjacency $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ s_{11} & s_{12} & s_{13} \end{bmatrix}$ to test for the stability of the matrix centre element a_{22}. By applying modified Sylvester's theorem check for positive definiteness.

The threshold chosen for normal images shall be $\pm 10\%$. But this threshold will vary based on the type of image and image modality.

$$a_{11} = s_{11} \pm T$$
$$a_{11}a_{22} + a_{12}a_{21} = (s_{11}s_0 + s_{12}s_{21}) \pm T$$
$$a_{11}\left(a_{22}s_{13} + a_{23}s_{12}\right) - a_{12}\left(a_{21}s_{13} + s_{11}a_{23}\right) + a_{13}\left(a_{21}s_{12} + a_{22}s_{11}\right)$$
$$= s_{11}\left(s_{22}s_{33} + s_{23}s_{32}\right) - s_{12}\left(s_{21}s_{33} + s_{31}s_{23}\right) + s_{13}\left(s_{21}s_{32} + s_{22}s_{31}\right) \pm T \quad (6)$$

The procedure for determining the stability of the pixel with the ROC is as follows.

i) If all the three conditions are satisfied, then the matrix considered is positive definite and hence the centre element is asymptotically stable which indicates that it belong to the region of convergence.

ii) If the first condition is not satisfied but the second and third condition is satisfied then the matrix considered is positive semi-definite and hence the centre element is marginally stable which indicates that it forms the edge layer between two distinguished regions

iii) If all three conditions are not satisfied then the matrix considered is negative definite and hence the centre element is unstable which indicates that it does not belong to the region of convergence.

iv) If the first condition is satisfied but the second or third condition is not satisfied then it indicates that the centre element of the matrix considered is a noisy pixel.

The procedure followed to check the stability of the centre element of region of adjacencies is illustrated.

$$\begin{bmatrix} & & a_{11} & a_{12} & a_{13} & & \\ & & a_{21} & a_{22} & a_{23} & & \\ b_{11} & b_{21} & s_{11} & s_{12} & s_{13} & d_{11} & d_{12} \\ b_{21} & b_{22} & s_{21} & s_0 & s_{23} & d_{21} & d_{22} \\ b_{31} & b_{32} & s_{31} & s_{32} & s_{33} & d_{31} & d_{32} \\ & & c_{11} & c_{12} & c_{13} & & \\ & & c_{21} & c_{22} & c_{23} & & \end{bmatrix} \quad \begin{bmatrix} & & a_{11} & a_{12} & a_{13} & & \\ & & a_{21} & [a_{22}] & a_{23} & & \\ b_{11} & b_{21} & s_{11} & s_{12} & s_{13} & d_{11} & d_{12} \\ b_{21} & [b_{22}] & s_{21} & s_0 & s_{23} & [d_{21}] & d_{22} \\ b_{31} & b_{32} & s_{31} & s_{32} & s_{33} & d_{31} & d_{32} \\ & & c_{11} & [c_{12}] & c_{13} & & \\ & & c_{21} & c_{22} & c_{23} & & \end{bmatrix}$$

$$\begin{bmatrix} e_{11} & e_{21} & a_{21} & & a_{23} & h_{11} & h_{12} \\ b_{11} & b_{21} & s_{11} & s_{12} & s_{13} & d_{11} & d_{12} \\ b_{21} & b_{22} & s_{21} & s_0 & s_{23} & d_{21} & d_{22} \\ b_{31} & b_{32} & s_{31} & s_{32} & s_{33} & d_{31} & d_{32} \\ f_{11} & f_{12} & c_{11} & & c_{13} & g_{11} & g_{12} \end{bmatrix} \quad \begin{bmatrix} e_{11} & e_{21} & a_{21} & & a_{23} & h_{11} & h_{12} \\ b_{11} & [b_{21}] & s_{11} & s_{12} & s_{13} & [d_{11}] & d_{12} \\ b_{21} & b_{22} & s_{21} & s_0 & s_{23} & d_{21} & d_{22} \\ b_{31} & [b_{32}] & s_{31} & s_{32} & s_{33} & [d_{31}] & d_{32} \\ f_{11} & f_{12} & c_{11} & & c_{13} & g_{11} & g_{12} \end{bmatrix}$$

$$\begin{bmatrix} i_{11} & a_{11} & a_{12} & a_{13} & l_{11} \\ e_{21} & a_{21} & a_{22} & a_{23} & h_{11} \\ b_{21} & s_{11} & s_{12} & s_{13} & d_{11} \\ & s_{21} & s_{0} & s_{23} & \\ b_{32} & s_{31} & s_{32} & s_{33} & d_{31} \\ f_{12} & c_{11} & c_{12} & c_{13} & g_{11} \\ j_{11} & c_{21} & c_{22} & c_{23} & k_{11} \end{bmatrix} \begin{bmatrix} i_{11} & a_{11} & a_{12} & a_{13} & l_{11} \\ e_{21} & [a_{21}] & a_{22} & [a_{23}] & h_{11} \\ b_{21} & s_{11} & s_{12} & s_{13} & d_{11} \\ & s_{21} & s_{0} & s_{23} & \\ b_{32} & s_{31} & s_{32} & s_{33} & d_{31} \\ f_{12} & [c_{11}] & c_{12} & [c_{13}] & g_{11} \\ j_{11} & c_{21} & c_{22} & c_{23} & k_{11} \end{bmatrix}$$

The same procedure is followed until a consecutive existence of unstable pixels is examined and all the elements that are marginally stable are joined together to obtain the segmented region. The proposed methodology for segmentation avoids the problem of over segmentation, good shape connectivity and avoids erroneous segmentation due to the presence of noisy pixel.

IV. RESULTS & DISCUSSIONS

The proposed methodology is applied for homogeneous images. The segmentation is performed with region of interest considered in different regions of the image. The different homogeneous regions in the images are segmented and by choosing different ROC's as with respect to the case or problem required. The results show that proper segmentation is obtained by the proposed method.

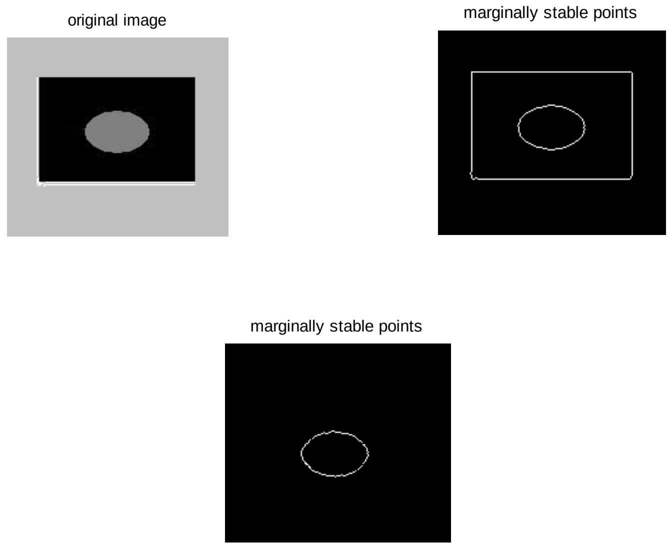

Fig 1. a) Original Image b) Segmentation of black regions c) Segmentation of inner gray regions

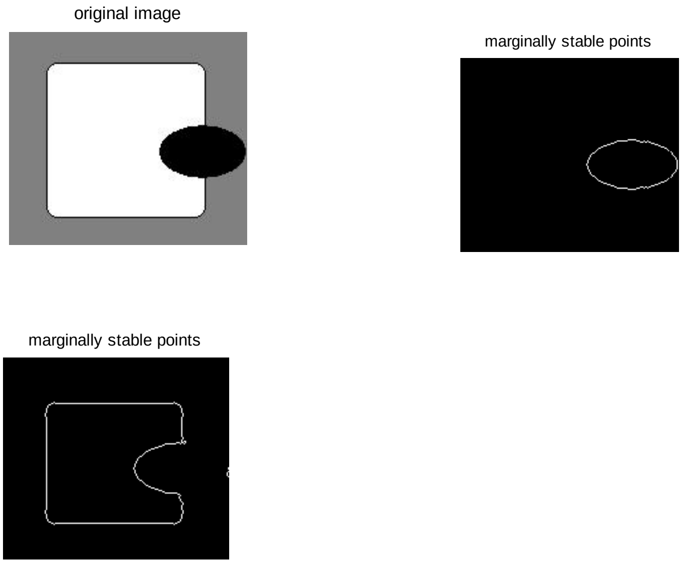

Fig 2. a) Original Image b) Segmentation of black region c) Segmentation of white region

11	2	23	14	15	21	15	15	13	13
14	11	13	16	13	16	22	16	12	12
13	12	10	12	248	251	253	13	15	14
12	18	255	252	255	253	252	248	14	15
14	249	252	254	22	251	255	246	13	10
238	254	253	251	250	252	254	251	253	249
245	248	252	249	255	255	256	253	252	253
248	245	251	248	254	256	255	250	16	251
252	252	249	247	255	255	253	249	254	250
254	254	247	250	253	251	252	247	252	251

Fig 3. Image pixel array for a certain region

The proposed method of segmentation is implemented on images with different homogeneous regions. The figures Fig 1 and Fig 2 shows the result of segmentation with the proposed method. The noisy pixels present in the region to be segmented are rid off and hence erroneous segmentation or over segmentation is avoided.

To show this let us consider the image pixel array of a certain region as shown in Fig 3. In the region consideredthere is an evidence of noisy pixel in two locations I (5, 5) and I (8, 9).

Since according to the fourth step in the procedure followed for segmentation, when there is noisy pixel the first condition is satisfied but second condition is not satisfied. Hence that particular pixel is not considered to be marginally stable but left out of segmentation

V. CONCLUSION & FUTURE WORK

A novel approach for image segmentation has been proposed based on the concept of stability. The results show that the process of segmentation is quicker and has good shape connectivity and good shape matching. Also the problem of over segmentation is not evident. Hence this method which is now modeled for normal images can be further improved to apply on heterogeneous medical images like MRI. Also the problem of proper choice in threshold for images like MRI has to be worked further to obtain better results.

REFERENCES

[1] J.Suri, D.Wilson, and S.Laxminarayanan, “Handbook of Biomedical Image Analysis”: Volume I,II: Segmentation Models. Kluwer Academic / Plenum Publishers, 2005

[2] S.Osher, J.A. Sethian,” Fronts propagating with Curvature dependent speed”, J Comp Physics, 79, 1988

[3] J.A.Sethian,”Level set methods and fast marching methods”, Cambridge University Press, 1999

[4] P.AYushkevich, J.Piven, H.C.Hazlett, R.G.Smith,S.Ho, J.C.Gee, G.Gerig,”User guided 3D active contour segmentation of anatomical structures: Significantly improved efficiency and reliability”, NeuroImage, 31(3), 2006, 1116-1128

[5] J.S.Suri, K.Liu, S.Singh, S.Laxminrayanan, X.Zeng ,L.Reden,”Shape Recovery algorithms using level sets in 2D/3D medical imagery:a state of the art review”, IEEE Transactions on Information Technology in Biomedicine, Vol.6, pp.8-28, 2002

[6] S.Yan, J.Yuan and C.Hou," Segmentation of medical ultrasound images based on level set method with edge representing mask", Advanced Computer Theory and Engineering, 3rd International Conference, Vol.2, 2010

[7] M.Strumia, D.Feltell, N.Evangelou, P.Gowland, C.Tench and L.Bai,"Gray matter segmentation of 7T MR images, IEEE Nuclear Science Symposium and Medical Imaging Conference, 2011, pp.3710-3714

[8] Katsuhiko Ogata, "Modern Control Engineering", Prentic Hall, 5th Edition, 2010

[9] M.Gopal, "Modern Control System Theory", New Age International Publishers, 1993

[10] Yixin Chen, James Z.Wang, Robert Krovetz, "Content Based Image Retrieval by Clustering",Proc of the 5th ACM SIGMM I'ntl workshop on Multimedia information retrieval ,New York,ACM press,pp-193-200,2003.

[11] Huiyu Zhou, Abdul H. Sadka, Mohammad R. Swash, JawidAzizi and Abubakar S. Umar., "Content Based Image Retrieval and Clustering: A Brief Survey" school of Engineering and Design, Brunel University, Uxbridge, UB8 3PH, UK

[12]Wang JZ, Li J, Wiederhold G. Simplicity: Semantics-sensitive integrated matching for picture libraries. IEEE Trans pattern Analysis Machine Intell;23:947-963, 2001.

[13] Jin J, Kurniawati R, Xu G, Bai X. Using browsing to improve content-based image retrieval. J Visual Common Image Represent;12:123-135, 2001.

[14] Huang Min,Sunbo,XiJianqing"An Optimized image retrieval method based on Hierarchical clustering and genetic algorithm"I'ntl forum on Information technology and applications,978-0-7695-3600-2/09-IEEE,2009