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Adaptive Synchronization between Different Chaotic Systems with Unknown Parameters

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Abstract

This article deals with synchronization between different chaotic systems such as Genesio-Tesi and Qi system using adaptive control method. Based on Lyapunov stability theory, the synchronization between a pair of chaotic systems with fully unknown parameters is derived. An adaptive control law and a parameter update rule for unknown parameters are designed such that the chaotic Qi system is controlled to be the chaotic Genesio-Tesi system. Numerical simulation results which are carried out using MATLAB, show that the adaptive control method is effective, easy to implement and reliable for synchronizing of the considered chaotic systems.

Keywords

Chaos, Synchronization, Genesio-Tesi system, Qi system, Adaptive control method.

1. Introduction

The applications of nonlinear dynamical systems have nowadays spread to a wide spectrum of disciplines including science, engineering, biology, sociology etc. Study and analysis of non-linear dynamics have gained immense popularity during the last few decades due to its important feature of any real-time dynamical system. In nonlinear systems, a small change in a parameter can lead to sudden and dramatic changes in both the qualitative and quantitative behavior of the system. Sometimes these may give rise to the complex behavior called chaos. Thus a chaotic system is a nonlinear deterministic system with unpredictable complexity. In dynamical systems, the term chaos is applied to deterministic systems that are aperiodic and that exhibit sensitive dependence on initial conditions and parameter variations,which is known as the butterfly effect [1]. The concept of chaos has been used to explain how systems that should be subject to known laws of physics may be predictable in the short term but are apparently random on a longer time scale.

In a chaotic synchronization, given a chaotic system, which is considered as the drive system, and another identical (or different) system, which is considered as the response system, the aim is to force the response system to synchronize the drive system.Since the idea of synchronizing chaotic systems was introduced by Pecora and Carroll [2] in 1990, theyshowed that it is possible to synchronize chaotic systems through a simple coupling. Synchronization of chaotic dynamical systems has been intensively studied by many researchers [3-6] and has attracted a great deal of interest in various field due to its important applications in ecological system [7], physical system [8], chemical system [9], modeling brain activity, system identification, pattern recognition phenomena and secure communications [10-12] etc.

In the recent years several different types of synchronization schemes have been proposed, such as time delay feedback approach [13], adaptive control [14-22], active control [23] sliding mode control [24] and so on, have been successfully applied to chaos synchronization. The concept of synchronization can be extended to complete synchronization [25], lag synchronization [26], phase synchronization [27], anti-synchronization [28], projective synchronization [29] etc.The control of chaotic systems is to design state feedback control laws that stabilizes the chaotic systems around the unstable equilibrium points. Adaptive control technique is used when the system parameters are unknown. In an adaptive method, control law and a parameter update rule for unknown parameters are designed in such a way that the chaotic response system is controlled to be the chaotic master system. The idea of synchronization is observed in periodic chaotic systems, which is a phenomenon in which the state variables of synchronized systems with different initial values have the same absolute values but opposite signs. The difference of two signals is expected to converge to zero when synchronization occurs. Recently, many authors have studied the synchronization for the chaotic systems. In 2005, Zhang et al. [14] studied adaptive synchronization between two different chaotic systems with unknown parameters. Hu et al. [15] proposed adaptive control for anti-synchronization of Chua's chaotic system.In 2008, Salarieh and Shahrokhi [16] have investigated adaptive synchronization of two different chaotic systems with time varying unknown parameters. In 2008, Wu [17] proposed adaptive synchronization between two different hyperchaotic systems. Zhang and Zhu [18] proposed anti-synchronization of two different hyperchaotic systems via active and adaptive control methods. Mossa et al. [19] have investigated adaptive anti-synchronization of chaotic systems with fully unknown parameters in 2010. Zhu and Cao [20] designed adaptive synchronization of chaotic Cohen–Crossberg neural networks with mixed time delays. Mossa et al. [21] have investigated adaptive anti-synchronization of two identical and different hyperchaotic systems with uncertain parameters in 2010. In 2011, Li et al. [22] proposed Complete (anti-)synchronization of chaotic systems with fully uncertain parameters by adaptive control.

In this article, an attempt has been taken to study synchronization between different chaotic systems using adaptive control method. This paper has been organized as follows. In Section 2, adaptive synchronization method is discussed. In section 3, the system descriptions of Genesio-TesiandQi systems are given. In Sections 4, adaptive synchronization between Genesio-Tesi and Qi chaotic systems is discussed. In Section 5, the conclusion of the work is presented.

2. Adaptive synchronization

Consider the drive chaotic system in the form of

$$\dot{x} = F(x) + \alpha\, f(x), \qquad (1)$$

where $x \in R^n$ is the state vector of the system, $\alpha \in R^m$ is the unknown parameter vector of the system, non linear term $F(x)$ is an $n \times 1$ matrix, $f(x)$ is an $n \times m$ matrix and the elements $f_{ij}(x)$ in the matrix $f(x)$ satisfy $f_{ij}(x) \in L_\infty$ for $x \in R^n$. On the other hand, the response system is assumed as

$$\dot{y} = G(x) + \beta g(x) + \mu(t), \quad (2)$$

where $y \in R^n$ is the state vector of the system, $\beta \in R^q$ is the unknown parameter vector of the system, non linear term $G(x)$ is an $n \times 1$ matrix, $g(x)$ is an $n \times q$ matrix, $\mu \in R^n$ is control input vector, and the elements $g_{ij}(x)$ in the matrix $g(x)$ satisfy $g_{ij}(x) \in L_\infty$ for $y \in R^n$. Let, $e = y - x$ be the error of the dynamical system for synchronization. The purpose of chaos synchronization is the proper design of the control parameter $\mu(t)$, such that, $\lim_{t\to\infty}\|e(t)\| = \lim_{t\to\infty}\|y(t, y_0) - x(t, x_0)\| = 0,$ where $\|\,.\,\|$ represents the Euclidean norm.

We subtract equation (1) from equation (2) and get

$$\dot{e} = -F(x) - \alpha f(x) + G(x) + \beta g(x) + \mu(t), \quad (3)$$

The parameters belonging to the drive and the response systems are always unknown. Therefore, by using the adaptive control and the parameter update rule techniques, the adaptive nonlinear controller can be selected as

$$\mu = F(x) + \hat{\alpha} f(x) - G(y) - \hat{\beta} g(y) - ke, \quad (4)$$

and adaptive laws of parameters are taken as

$$\dot{\hat{\alpha}} = [f(x)]^T e, \; \dot{\hat{\beta}} = [g(y)]^T e \quad (5)$$

then the response system (2) can synchronize the drive system (1) globally and asymptotically, where k > 0 is a constant, $\hat{\alpha}$ and $\hat{\beta}$ are, respectively, estimations of the unknown parameters α and β, where α and β are constants.

Assume a positive Lyapunov function

$$V = \frac{1}{2}[e^T e + \overline{\alpha}^T \overline{\alpha} + \overline{\beta}^T \overline{\beta}],$$ where

$$\overline{\alpha} = (\alpha - \hat{\alpha}), \; \overline{\beta} = (\beta - \hat{\beta}).$$

With the choice of the adaptive control law and parameter update rule above for unknown parameters are designed, the time derivative of V along the solution in equation (3) will be smaller than zero. In other words, the error vector will approach to zero as time goes infinite and from Lyapunov stability theory , the states of the slave system and master system are asymptotically synchronized. This implies that the trajectory of the response system (2) with initial condition y_0 can asymptotically approaches to the drive system (1) with initial condition x_0 and finally implements the synchronization.

3. Systems' descriptions

3.1 Genesio-Tesi system

TheGenesio–Tesi chaotic system [30] is one of paradigms of chaos since it captures many features of chaotic systems. It includes a simple square part and three simple ordinary differential equations that depend on three positive real parameters. The dynamic equation of the system is as follows

$$\begin{aligned} D_t x &= y \\ D_t y &= z \\ D_t z &= -a x - b y - c z + m x^2, \end{aligned} \quad (6)$$

where x, y, z are state variables, and a, b and c are the positive real constants satisfying $cb < a$ and parameters are $a = 6,\ b = 2.92,\ c = 1.2$ and $m = 1$ yield chaotic trajectory. The chaotic attractors in x-y-z space and x-y, y-z, z-x planes are depicted through Fig. 1.

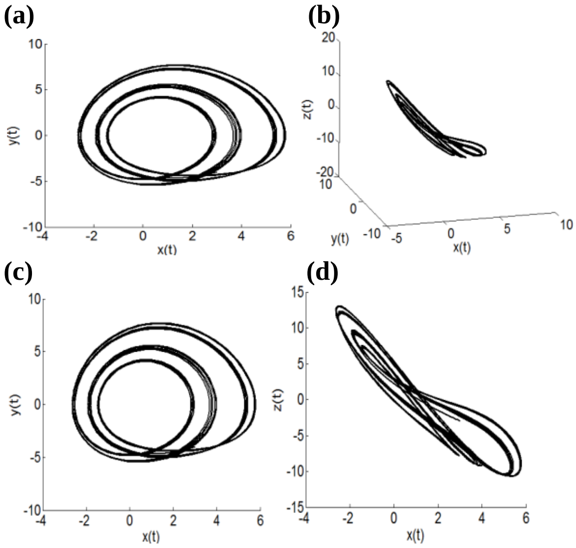

Fig.1. Phase portraits of Genesio-Tesi chaotic attractor in (a) *x-y-z* space(b) x-y plane (c)*y-z* plane and (d)*x-z* plane.

The Qi chaotic system [31] is described by

$$\begin{aligned} D_t x &= p(y - x) + yz \\ D_t y &= r x - y - x z \\ D_t z &= -q z + x y, \end{aligned} \quad (7)$$

where $p = 35,\ q = 8/3,\ r = 80$ yield chaotic trajectory. The chaotic attractors in x-y-z space and x-y, y-z, z-x plane are depicted through Fig.2.

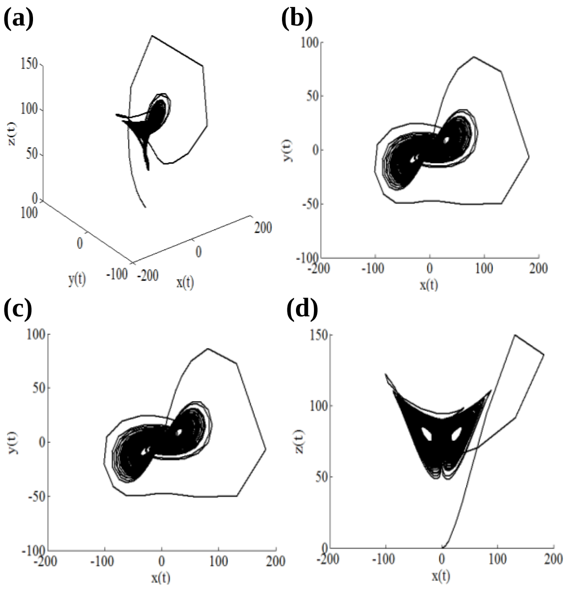

Fig.2. Phase portraits of Qi chaotic attractor in (a) x-y-z space(b) x-y plane (c)y-z plane and (d)x-z plane.

4. Adaptive synchronization between Genesio-Tesi and Qi chaotic systems

In this section, the synchronization between Genesio-Tesi chaotic system (8) and chaotic Qi system (9) are studied, we assume that Genesio-Tesi system with four unknown parameters drives the Qi system with three unknown parameters.

The drive system is given by

$$\begin{aligned} D_t x_1 &= y_1 \\ D_t y_1 &= z_1 \\ D_t z_1 &= -a x_1 - b y_1 - c z_1 + m x_1^2 \end{aligned} \qquad (8)$$

The response system is described by

$$\begin{aligned} D_t x_2 &= p(y_2 - x_2) + y_2 z_2 + \mu_1(t) \\ D_t y_2 &= r x_2 - y_2 - x_2 z_2 + \mu_2(t) \\ D_t z_2 &= -q z_2 + x_2 y_2 + \mu_3(t), \end{aligned} \qquad (9)$$

where $\mu(t) = [\mu_1(t),\ \mu_2(t),\ \mu_3(t)]^T$ are three control functions to be designed. In order to determine the control functions to realize the synchronization between systems (8) and (9), we subtract equation (9) to (8),

$$\begin{aligned} D_t e_1 &= p(y_2 - x_2) + y_2 z_2 - y_1 + \mu_1(t) \\ D_t e_2 &= r x_2 - x_2 z_2 - y_2 - z_1 + \mu_2(t) \\ D_t e_3 &= a x_1 + b y_1 + c z_1 - m x_1^2 - q z_2 + x_2 y_2 + \mu_3(t), \end{aligned} \qquad (10)$$

where $e_1 = x_2 - x_1,\ e_2 = y_2 - y_1,\ e_3 = z_2 - z_1$. Our main aim is to find proper control functions $\mu_i(t),\ i = 1,2,3$ and parameter update rule, such that system (9) globally synchronizes system (8) asymptotically , i.e. $\lim_{t\to\infty} \|e(t)\| = 0.$

For two systems (8) and (9) without controls $(\mu_i(t) = 0, i = 1,2,3),$ if the initial condition $(x_1(0), y_1(0), z_1(0)), \neq (x_2(0), y_2(0), z_2(0)),$ then the trajectories of two systems will quickly separate each other and become irrelevant. However, when controls are applied, the two systems will approach synchronization for any initial conditions by appropriate control functions. With this idea, we propose the following adaptive control law for system equation (9)

$$\begin{aligned} \mu_1(t) &= -\hat{p}(y_2 - x_2) - y_2 z_2 + y_1 - k_1 e_1 \\ \mu_2(t) &= -\hat{r} x_2 + x_2 z_2 + y_2 + z_1 - k_2 e_2 \\ \mu_3(t) &= -\hat{a} x_1 - \hat{b} y_1 - \hat{c} z_1 + \hat{m} x_1^2 + \hat{q} z_2 - x_2 y_2 - k_3 e_3, \end{aligned} \qquad (11)$$

and parameters update rule for seven unknown parameters $a,$ $b,\ c,\ m,\ p,\ q,\ r$

$$\begin{cases} \dot{\hat{a}} = x_1 e_3,\ \dot{\hat{b}} = y_1 e_3,\ \dot{\hat{c}} = z_1 e_3,\ \hat{m} = -x_1^2 e_3 \\ \dot{\hat{p}} = (y_2 - x_2) e_1,\ \dot{\hat{q}} = -z_2 e_3,\ \dot{\hat{r}} = x_2 e_2, \end{cases} \qquad (12)$$

where $k_i\ (i = 1,2,3)$ are positive real scalars and $\hat{a},\ \hat{b},\ \hat{c},$ $\hat{m},\ \hat{p},\ \hat{q}\ \hat{r}$ are estimates values of $a,\ b,\ c,\ m,\ p,\ q,\ r$ respectively.

Theorem 1. For any initial conditions, the two systems (8) and (9) are globally asymptotically synchronized by adaptive control law (11) and parameter update rule (12).

Proof. Equation (11) with equation (10) yields the error dynamics as

$$\begin{aligned} D_t e_1 &= \bar{p}(y_2 - x_2) - k_1 e_1 \\ D_t e_2 &= \bar{r} x_2 - k_2 e_2 \\ D_t e_3 &= \bar{a} x_1 + \bar{b} y_1 + \bar{c} z_1 - \bar{m} x_1^2 - \bar{q} z_2 - k_3 e_3, \end{aligned} \qquad (13)$$

where $\bar{a} = a - \hat{a},\quad \bar{b} = b - \hat{b},\quad \bar{c} = c - \hat{c},$ $\bar{m} = m - \hat{m},\ \bar{p} = p - \hat{p},\ \bar{q} = q - \hat{q},\ \bar{r} = r - \hat{r}$

Consider the following Lyapunov function

$$V(t) = \frac{1}{2}(e^T e + \bar{a}^2 + \bar{b}^2 + \bar{c}^2 + \bar{m}^2 + \bar{p}^2 + \bar{q}^2 + \bar{r}^2). \quad (14)$$

The time derivative of V along the solution of error dynamical systems gives that

$$\begin{aligned} \dot{V}(t) &= e^T \dot{e} + \bar{a}\dot{\bar{a}} + \bar{b}\dot{\bar{b}} + \bar{c}\dot{\bar{c}} + \bar{m}\dot{\bar{m}} + \bar{p}\dot{\bar{p}} + \bar{q}\dot{\bar{q}} + \bar{r}\dot{\bar{r}} \\ &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + \bar{a}(-\dot{\hat{a}}) + \bar{b}(-\dot{\hat{b}}) + \bar{c}(-\dot{\hat{c}}) \\ &\quad + \bar{m}(-\dot{\hat{m}}) + \bar{p}(-\dot{\hat{p}}) + \bar{q}(-\dot{\hat{q}}) + \bar{r}(-\dot{\hat{r}}) \\ &= e_1[\bar{p}(y_2 - x_2) - k_1 e_1] + e_2[\bar{r} x_2 - k_2 e_2] + \\ &\quad e_3[\bar{a} x_1 + \bar{b} y_1 + \bar{c} z_1 - \bar{m} x_1^2 - \bar{q} z_2 - k_3 e_3] \\ &\quad + \bar{a}(-x_1 e_3) + \bar{b}(-y_1 e_3) + \bar{c}(-z_1 e_3) + \\ &\quad \bar{m}(x_1^2 e_3) + \bar{p}(-(y_2 - x_2) e_1) + \bar{q}(z_2 e_2) \\ &\quad + \bar{r}(-x_2 e_2) \\ &= -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \\ &= -e^T P e \le 0, \end{aligned} \qquad (15)$$

where

$$e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}, \quad P = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix}.$$

Since $\dot{V}$ is negative semi definite. Then $e_1, e_2, e_3, \hat{a}, \hat{b}, \hat{c}, \hat{m}, \hat{p}, \hat{q}, \hat{r} \in L_\infty$. From error system (13), we have $\dot{e}_1, \dot{e}_2, \dot{e}_3 \in L_\infty$. Since $\dot{V}(t) = -e^T P e$ and P is a positive definite matrix, then we have

$$\int_0^t \lambda_{\min}(P)\|e\|^2 \, dt \le \int_0^t e^T P e \, dt = \int_0^t -\dot{V} \, dt = \qquad (16)$$
$$= V(0) - V(t) \le V(0),$$

where, $\lambda_{\min}(p)$ is the minimum eigen value of positive-definite matrix P .Thus $\dot{e}_1, \dot{e}_2, \dot{e}_3 \in L_2$. According to the Barbalatslemma,we have $e_1(t), e_2(t), e_3(t) \to 0$ as $t \to \infty$.
Therefore, response system (9) can globally synchronized the drive system (8) asymptotically. This completes the proof.

4.1 Numerical simulations and results

In numerical simulations, the parameters of Genesio-Tesi and Qi systems are taken as $(a,b,c,m) = (6, 2.92, 1.2, 1)$ and $(p,q,r) = (35, 8/3, 80)$ respectively, such that both the systems exhibit chaotic behavior. The initial values of the drive and response systems are taken as $(x_1(0), y_1(0), z_1(0)) = (3, -4, -13)$ and $(x_2(0), y_2(0), z_2(0)) = (12, -10, 2)$ respectively. Thus, the initial errors will be $(e_1(0), e_2(0), e_3(0)) = (9, -6, 15)$. The fourth order Range-Kutta method is used to solve the two systems of equations (8) and (9) with time step size is taken as 0.001. We assume that control inputs $(k_1, k_2, k_3) = (1, 1, 1)$. Synchronization of systems (8) and (9) via adaptive control laws (11) and parameter update rule (12) with the initial estimated parameters $(\hat{a}(0), \hat{b}(0), \hat{c}(0), \hat{m}(0)) = (2, -3, -5, -1)$ and $(\hat{p}(0), \hat{q}(0), \hat{r}(0)) = (-4, 6, 2)$ are shown in Fig.3 and Fig.4. Fig.3 shows the state response and the synchronization error system (13) converges to zero and Fig.4 shows that the estimated values $(\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{m}(t))$ and $(\hat{p}(t), \hat{q}(t), \hat{r}(t))$ of unknown parameters of the systems (8) and (9) converges to $(a, b, c, m) = (6, 2.92, 1.2, 1)$ and $(p, q, r) = (35, 8/3, 80)$ respectively as $t \to \infty$.

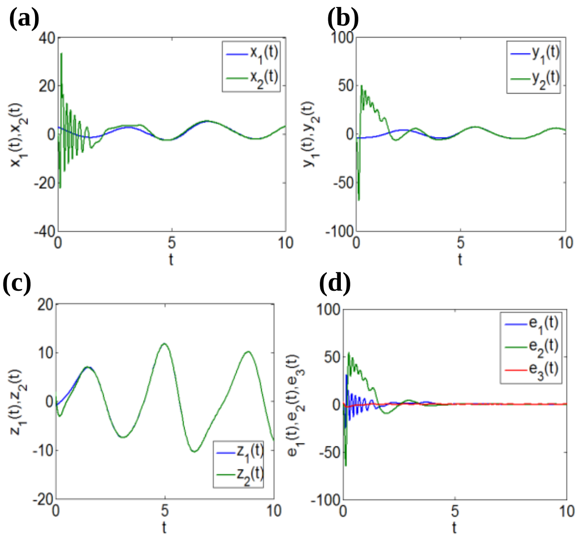

Fig.3. Adaptive synchronization of drive system (8) and response system (9): (a) between $x_1 - x_2$ signals(b) between $y_1 - y_2$ signals(c) between $z_1 - z_2$ signals and (d) The error functions $e_1(t), e_2(t)$ and $e_3(t)$ with time t.

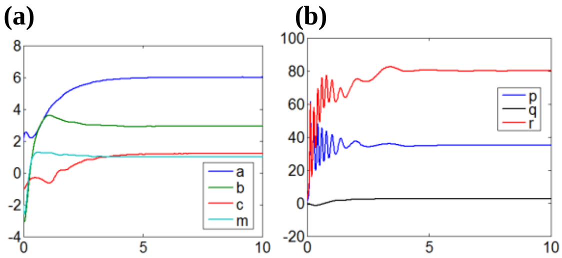

Fig.4. Estimate values of parameters (a) a, b, c, m and (b) p, q, r of Genesio-Tesi and Qi systems with parameter update rule (12).

6. Conclusion

The present investigation has attained accomplishment in two significant capacities. First it successfully carried out the study of synchronization between Genesio-Tesi and Qi chaotic systems with uncertain parameters using adaptive control method. Adaptive controller and parameters update law are designed properly to synchronize two different pair of chaotic systems based on the Lyapunov stability theorem. The second one is the numerical simulation, which are carried out using Runge-Kutta method calls for appreciation to show that the method is reliable and effective for adaptive synchronization of nonlinear dynamical systems.

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