Poisson Reducing Unilateral Filtering for X-ray Image Denoising

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ABSTRACT
This paper is enhancement of author’s earlier work, Poisson noise Reducing Bilateral Filter (PRBF). This paper recommends two major changes in PRBF. One change is to make PRBF independent of distance variance i.e. filter performance is based on single parameter (range variance). Therefore, this proposed work is named as Poisson Reducing Unilateral Filtering (PRUF). Similarly, performance of PRBF on edge region is enhanced due to second change and same is demonstrated through experimentation. Peak signal to noise ratio (PSNR) and Structural similarity index matching (SSIM) quality metrics are used for comparison of proposed PRUF with existing PRBF.

General Terms
Image denoising, Poisson noise reducing bilateral filter, X-ray images, Bilateral filter.

Keywords
Poisson noise, X-ray denoising, Bilateral filter, Poisson noise reducing bilateral filter, PSNR, SSIM.

1. INTRODUCTION
More or less image noise is observed in each type of images. X-rays, Ultrasound, CT-Scan and MRI are examples of medical images. These are also known as digital modalities in medical imaging. Image acquisition mechanism is different for different modalities. Particularly image acquisition process is responsible for formation of noise and hence type of noise present in these images is distinct. Different noise carry distinguishable characteristics for example speckle noise is multiplicative in nature and Poisson noise is signal dependent etc. Out of this, focus of this paper is on X-ray images. X-ray imaging is a popular digital modality mainly to detect fractures in bone, blood flow etc. at low cost. Earlier this modality was analog but nowadays most of X-ray scanners are digital in nature. Digitization in X-ray imaging gives fast results compared to traditional analog X-ray. This digital X-ray gives better visual results than that of earlier techniques.

In spite of this, X-ray images suffer from Poisson noise and occasionally results into misleading decisions. To overcome this issue, researchers offer variety of algorithms in image domain, sometimes in transform domain or even in hybrid domain. Most of the algorithms viz. [2] and [3] are based on assumption that image noise is Gaussian noise but this is not true for every type of images. X-ray is medical image and it has Poisson noise dominance. To handle this kind of noise, one can preprocess X-ray image with variance stabilization transformation or similar kind of transform to change Poisson distribution into Gaussian distribution and enjoy benefits of traditional algorithms. Reference [7] is example of above stated approach. Reference [3] introduced BM3D technique for Gaussian noise reduction. This is the current state of the art method and modified in [7] for Poisson noise reduction. This is a hybrid method based on Anscombe transformation. Authors in reference [1] proposed hybrid method using Shearlet transform and bilateral filtering for medical images. Authors in reference [6] proposed hybrid method using Poisson reducing bilateral filter for Gaussian noise removal using two types of weights. These weights consider geometric distance and photometric/intensity distance. Principal advantage of this algorithm is edge preservation along with simplicity. This algorithm was further modified for speckle noise reduction and named as Speckle reducing bilateral filter (SRBF) in reference [8]. Similarly, bilateral filter is modified for Poisson noise by transforming photometric distance weight according to Poisson distribution in reference [4]. Authors introduced this algorithm as PRBF filter in spatial domain. The main goal of this paper is to enrich the performance of this PRBF.

Rest of the paper is organized in section 2, 3, 4 and 5. Section 2 clarifies brief theory of bilateral filter and Poisson reducing bilateral filter. Section 3 describes proposed work for noise reduction. Implementation and experimentation details are specified in the section 4. Paper is concluded with section 5.

2. THEORY OF BILATERAL FILTER
Bilateral filter [1] performs filtering operation by considering geometric distance and photometric distance of pixels in local neighborhood. As per reference [1], the main application of bilateral filter is Gaussian noise removal and preserving edges of input image. Bilateral filter modifies candidate pixel value as per following equation (1),

\[ J(i, j) = W * I(i, j)/\text{sum}(W) \]  

(1)

\[ W = G * H \]  

(2)

Where, \( J(i, j) \) = processed pixel value; \( I(i, j) \) = input pixel value, \( W \) = weight matrix calculated by combination of geometric and photometric weight matrices in corresponding local window, \( G \) = the geometric weight matrix and \( H \) = the photometric weight matrix.

To calculate these weight matrices in the local window, authors in [1] considered Gaussian distribution to remove the Gaussian noise from the image. In previous work [4], authors modified this bilateral filter to remove Poisson noise from X-ray images. In that approach, photometric weight is calculated by considering Poisson distribution as given in equation (3)

\[ P(X = k) = e^{-\lambda} \frac{\lambda^k}{k!} \]  

(3)

Here, \( \lambda \) is considered as mean of local window and \( k \) is estimated from Maximum Likelihood Estimation (MLE) as
per reference [5]. From MLE, k is same as mean of local window that is k = \lambda. This is interpreted as k is fixed for local window with mean value [4].

3. PROPOSED WORK

Despite some advantages such as simplicity, non-iterative and edge preserving nature, there is drawback associated with bilateral filter, which is selection of parameters. These parameters include geometric, photometric variances and window size. These parameters are decided by user. Depending upon these parameters, performance of the bilateral filter changes tremendously. To overcome this limitation, authors proposed Poisson noise reducing spatial domain filter named as Poisson Reducing Unilateral Filter (PRUF). It does not consider geometric distances but considers only distribution of intensities in local window.

From literature, it is clear that X-ray image formation follows Poisson statistics and hence noise also follows same distribution. Taking into account this fact, weight matrix is calculated by Poisson distribution formula (mentioned in equation (3)) in proposed algorithm. To evaluate parameter ‘\lambda’ following few steps are used.

Mean of small set of numbers is calculated as,

\[ \text{Mean} = \sum_{i=0}^{n} x_i p_i \]  

(4)

Here, \(x_i\) is the corresponding number and \(p_i\) is respective probability value. In short, \(p\) is probability distribution function for that set of numbers, with sum of all probabilities equals one. In our case, intensities in local window of X-ray images follow Poisson distribution. Hence, in above equation (4), \(p_i\) is replaced by Poisson distribution formula. By substituting equation (3) in equation (4), it is re-written as,

\[ \text{Mean} = \sum_{i=1}^{n} x_i e^{-\lambda} \frac{\lambda^x}{x!} \]  

(4-A)

\[ \text{Mean} = \lambda \sum_{i=0}^{n} e^{-\lambda} \frac{\lambda^x}{(\lambda-1)^x} \]  

(4-B)

In above equation, by putting another variable \(y\) instead of \((x-1)\) results into same probability distribution function as that of equation (3)

\[ \text{Mean} = \lambda \sum_{i=1}^{n} e^{-\lambda} \frac{\lambda^y}{y!} = \lambda \sum p_i \]  

(4-C)

Since, sum of all probabilities is equal to one, above equation reduces to equation (5) as,

\[ \text{Mean} = \lambda \]  

(5)

Interpretation of equation (5) for proposed work is that parameter \(\lambda\) is nothing but mean of intensities in local window.

Algorithm for proposed work is as follows,

Step 1: Give the noisy X-ray image to the proposed filter as input.

Step 2: Form overlapping local window from the input noisy image as,

Row_Min = max (i-w, 1); Row_Max = min (i+w, M);

Col_Min = max (j-w, 1); Col_Max = min (j+w, N);

I = A (Row_Min: Row_Max, Col_Min: Col_Max);

Here, ‘w’ is window size; ‘M’ and ‘N’ are total number of rows and columns in the image. ‘A’ is input noisy image and ‘I’ is the local window extracted from given noisy image. A (i, j) is the pixel whose value is to be modified.

Step 3: For local window ‘I’ extracted from given noisy image, calculate \(\lambda\) as the mean of that local window.

Step 4: For each pixels in the local window ‘I’, calculate weight by considering Poisson distribution as

\[ W = P(X = k) = e^{-\lambda} \frac{\lambda^k}{k!} \]  

(6)

Here, ‘k’ is the individual pixel intensity in the local window. Calculation of factorial is a cumbersome, time consuming process as the input number goes on increasing. Hence, instead of factorial, authors used Stirling’s approximation of the given intensity value in this paper. Stirling’s approximation gives faster output than that of factorial. Stirling’s approximation of a number ‘n’ is given by,

\[ n! \sim \sqrt{2\pi n} \left(\frac{n}{e}\right)^n \]  

(7)

Where, \(n = 3.1412; e = 2.7182;\)

Step 5: Calculation of weights forms a matrix of same size as that of local window ‘I’. According to these weights, modified corresponding pixel is given as,

\[ B(i, j) = \frac{W*(i, j)}{\sum(W)} \]  

(8)

Here, \(B(i, j)\) is reconstructed pixel value, I (i, j) is input noisy pixel and \(W\) is the weight calculated by equation (6).

Step 6: Repeat step 2 to step 5 to reconstruct the Poisson noise corrupted image.

As compared to author’s previous work [4], two major changes are made in this proposed work those are:

1) Geometric filtering is not used. This results into reduction of input parameters required in the filter. Hence performance of filter becomes more reliable as compared to previous filters mentioned in references [1 and 4].

2) In weight calculation, individual pixel intensity is considered. Though this increases computational time, enrichment in edge preservation capability is observed.

Because of these modifications, visual quality of images is improved for X-ray images. First point is observed in section 4 and second point is explained by taking synthetically generated image as shown in figure 1 below.

Here, synthetic image of size 8 by 8 is considered with two intensity levels as black and white. Poisson noise is added in that synthetically generated image and image is reconstructed by using both PRBF and proposed filter. Figure 1 shows noisy and filtered images along with their intensity values. This figure clearly shows that proposed Poisson reducing unilateral filter (PRUF) outperforms previous filter in the context of edge preservation.
Fig. 1: (a) Poisson noise corrupted synthetic image with intensity values, (b) Filtered image by PRBF with intensity values and (c) Filtered image by proposed method with intensity values.

4. RESULTS AND DISCUSSION

For validation of proposed algorithm and comparing the results with previous methods, MATLAB 13a software is used. First, proposed method is applied to MATLAB test images such as Cameraman, Peppers, etc. Since Poisson noise is signal dependent noise, to change the noise variance in the image, peak intensity of the image is changed from 10, 20 etc. Hence, for each image, three different image intensities are considered.

Table 1 depicts the comparison of general purpose images (Cameraman and Peppers) with algorithms [2], [1] and [4] with respect to Peak Signal to Noise Ratio (PSNR) quality metric.

Table 1. PSNR Comparison for General Images

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cameraman</td>
<td>10</td>
<td>13.28</td>
<td>25.07</td>
<td>19.39</td>
<td>21.04</td>
<td>21.06</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>18.03</td>
<td>27.42</td>
<td>21.18</td>
<td>23.69</td>
<td>23.58</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>24.05</td>
<td>29.47</td>
<td>24.81</td>
<td>25.28</td>
<td>25.36</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>17.91</td>
<td>28.07</td>
<td>24.78</td>
<td>25.51</td>
<td>25.91</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>23.92</td>
<td>31.06</td>
<td>26.80</td>
<td>29.41</td>
<td>30.33</td>
</tr>
</tbody>
</table>

Actually, PSNR is given by following equation (9)

$$PSNR = 10 \log_{10} \left( \frac{2^n - 1}{MSE} \right)$$  \hspace{1cm} (9)

Where \( n = \) no. of bits and \( MSE = \) Mean Square Error

MSE is evaluated by equation (10)

$$MSE = \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} (X_{ij} - Y_{ij})^2$$  \hspace{1cm} (10)

It is observed from equation (9) that PSNR quality metric considers image size only. Another metric SSIM is proposed in reference [9]. Formula for calculation of SSIM quality metric is given below in equation (11),

$$SSIM = \frac{f(l(x, y)c(x, y)s(x, y))}{\sqrt{f(l(x, y))f(c(x, y))f(s(x, y))}}$$  \hspace{1cm} (11)

Where, \( f(l(x, y)c(x, y)s(x, y)) \) gives luminance, contrast and structural comparison respectively between two images \( x \) and \( y \). Following table 2 provides comparison of proposed algorithm with earlier PRBF [4]. Here quality metric SSIM is calculated for different X-ray images.

Table 2. Comparison of proposed work with PRBF [4]

<table>
<thead>
<tr>
<th>Image</th>
<th>Peak Intensity</th>
<th>Noisy SSIM</th>
<th>PRBF SSIM</th>
<th>Prop. Method SSIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>X2</td>
<td>20</td>
<td>0.9658</td>
<td>0.9873</td>
<td>0.9930</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>0.9380</td>
<td>0.9760</td>
<td>0.9855</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>0.9150</td>
<td>0.9660</td>
<td>0.9766</td>
</tr>
<tr>
<td>X4</td>
<td>20</td>
<td>0.9528</td>
<td>0.9828</td>
<td>0.9924</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>0.9127</td>
<td>0.9671</td>
<td>0.9853</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>0.8791</td>
<td>0.9528</td>
<td>0.9779</td>
</tr>
<tr>
<td>Chest</td>
<td>20</td>
<td>0.9578</td>
<td>0.9845</td>
<td>0.9916</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>0.9258</td>
<td>0.9771</td>
<td>0.9862</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>0.8995</td>
<td>0.9607</td>
<td>0.9807</td>
</tr>
<tr>
<td>Knee</td>
<td>20</td>
<td>0.8949</td>
<td>0.9857</td>
<td>0.9915</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>0.8309</td>
<td>0.9741</td>
<td>0.9862</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>0.7866</td>
<td>0.9636</td>
<td>0.9798</td>
</tr>
<tr>
<td>Spine</td>
<td>20</td>
<td>0.8874</td>
<td>0.9849</td>
<td>0.9930</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>0.8084</td>
<td>0.9729</td>
<td>0.9896</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>0.7460</td>
<td>0.9630</td>
<td>0.9850</td>
</tr>
</tbody>
</table>

It is observed from above table 2 that proposed work outperform as compared to Poisson noise Reducing Bilateral filter. Same performance is observed for general images in...
Again not only quality metrics improvement is observed but enhanced visual quality is also obtained with proposed PRUF filter. Some sample visible results are displayed below for ready reference.

Figure 2, 3 and 4 are set of noisy image, PRBF result and proposed filter’s results for chest X-ray, knee X-ray and spine X-ray image respectively.

From visible results, it is observed that performance of proposed Poisson Reducing Unilateral Filter really works better than Poisson noise reducing bilateral filter w.r.t. numerical quality metrics as well as visible results.

5. CONCLUSIONS AND FUTURE SCOPE

At the end, authors conclude that PRUF is advanced version of PRBF, principally designed to denoise medical X-ray images. Performance of PRBF at edges is improved by evaluating instantaneous value of \( k \) in local window. For implementation ease factorial is replaced by Stirling approximation. Similarly, PRUF is independent of distance variance i.e. reliability of filter is more than earlier PRBF.
In future, performance of PRUF filter could be improved by considering specific behavior of Poisson noise.

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7. REFERENCES


