International Journal of Applied Information Systems (IJAIS) – ISSN : 2249-0868
Foundation of Computer Science FCS, New York, USA
Volume 8– No.4, February 2015 – www.ijais.org

3-Total Super Sum Cordial Labeling for Union of Some Graphs

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ABSTRACT
In this paper we investigate 3-total super sum cordial labeling for union of some graphs. Suppose $G = (V(G), E(G))$ be a graph with vertex set $V(G)$ and edge set $E(G)$. A vertex labeling $f: V(G) \to \{0,1,2\}$. For each edge uv assign the label$(f(u) + f(v)) mod\ 3$. The map f is called a 3-total super sum cordial labeling if $|f(i) - f(j)| \leq 1$for $i, j\ \varepsilon\ \{0,1,2\}$ where $f(x)$ denotes the total number of vertices and edges labeled with $x = \{0,1,2\}$ and for each edge $uv, |f(u) - f(v)| \leq 1$. Any graph which satisfies 3-total super sum cordial labeling is called 3-total super sum cordial graphs. Here we prove some graphs like $P_m \cup P_n, C_m \cup C_n, k_1, m \cup k_1, n$ are 3-total super sum cordial graphs.

Keywords
3-total super sum cordial labeling, 3-total super sum cordial graphs.

1. INTRODUCTION
The graphs considered here are finite, undirected an simple. The vertex set and edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. Graphs $G_1$and G_2 have disjoint point set V_1 and V_2 and edge sets E_1 and E_2 respectively. The union of G_1 and G_2 is the graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$. The concept of sum cordial labeling of graph was introduced in [5] and that of k-sum cordial labeling in [4]. The concept of 3-total sum cordial labeling of a graph was introduced in [6].

Definition 1.1.: Let G be a graph. Let f be a map from $V(G)$to $\{0,1,2\}$. For each edge uv assign the label $(f(u) +$ *fv(mod 3)*. Then the map f is called 3-total sum cordial labeling of G, if $|f(i) - f(j)| \leq 1$: $i, j\ \varepsilon\ \{0,1,2\}$ where $f(x)$ denotes the total number of vertices and edges labeled with $x = \{0,1,2\}$.

Definition 1.2.: A 3-total sum cordial labeling of a graph G is called 3-total super sum cordial labeling if for each edge $uv\ |f(u) - f(v)| \leq 1$. A graph G is 3-total super sum cordial if it admits 3-total super sum cordial labeling.

2. PRELIMINARIES
Theorem 2.1.: $P_m \cup P_n$ is 3-total super sum cordial.

Proof: Let P_m be the path $u_1, u_2, \ldots, u_m$ and P_n be the path $v_1, v_2, \ldots, v_n$

Case I: $m \equiv 0\ (mod\ 3)$ and $n \equiv 0\ (mod\ 3)$

Let $m = 3p$ and $n = 3t$

Assign

$$f(u_1) = 0$$
$$f(u_2) = 1$$
$$f(u_3) = 2$$

Define

$$f(u_{3i+4}) = 1; \quad 0 \leq i < p - 1$$
$$f(u_{3i+5}) = 1; \quad 0 \leq i < p - 1$$
$$f(u_{3i+6}) = 2; \quad 0 \leq i < p - 1$$

Assign

$$f(v_n) = 1$$
$$f(v_{n-1}) = 1$$
$$f(v_{n-2}) = 2$$

Define

$$f(v_{3i+1}) = 2; \quad 0 \leq i < t - 1$$
$$f(v_{3i+2}) = 1; \quad 0 \leq i < t - 1$$
$$f(v_{3i+3}) = 2; \quad 0 \leq i < t - 1$$

Hence f is 3-total super sum cordial

Case II: $m \equiv 0\ (mod\ 3)$ and $n \equiv 1\ (mod\ 3)$

Let $m = 3p$ and $n = 3t + 1$

Define

$$f(u_{3i+1}) = 2; \quad 0 \leq i < p$$
$$f(u_{3i+2}) = 2; \quad 0 \leq i < p$$
$$f(u_{3i+3}) = 1; \quad 0 \leq i < p$$

Assign;

$$f(v_n) = 1$$

Define

$$f(v_{3i+1}) = 2; \quad 0 \leq i < t$$
$$f(v_{3i+2}) = 1; \quad 0 \leq i < t$$
$$f(v_{3i+3}) = 2; \quad 0 \leq i < t$$

Hence f is 3-total super sum cordial.

Case III: $m \equiv 0\ (mod\ 3)$ and $n \equiv 2\ (mod\ 3)$

Let $m = 3p$ and $n = 3t + 2$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \leq i < p$$

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$$f(u_{3i+2}) = 1; \quad 0 \le i < p$$
$$f(u_{3i+3}) = 2; \quad 0 \le i < p$$

Assign:

$$f(v_n) = 1$$
$$f(v_{n-1}) = 2$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i \le t-1$$
$$f(v_{3i+2}) = 1; \quad 0 \le i \le t-1$$
$$f(v_{3i+3}) = 2; \quad 0 \le i \le t-1$$

Hence f is 3-total super sum cordial.

Case IV: $m \equiv 1\ (mod\ 3)$ and $n \equiv 0\ (mod\ 3)$

Let $m = 3p+1$ and $n = 3t$

Assign:

$$f(u_m) = 1$$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p$$
$$f(u_{3i+2}) = 1; \quad 0 \le i < p$$
$$f(u_{3i+3}) = 2; \quad 0 \le i < p$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i < t$$
$$f(v_{3i+2}) = 2; \quad 0 \le i < t$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t$$

Hence f is 3-total super sum cordial labeling.

Case V: $m \equiv 1\ (mod\ 3)$ and $n \equiv 1\ (mod\ 3)$

Let $m = 3p+1$ and $n = 3t+1$

Assign:

$$f(u_m) = 2$$

Define

$$f(u_{3i+1}) = 2; \quad 0 \le i < p$$
$$f(u_{3i+2}) = 1; \quad 0 \le i < p$$
$$f(u_{3i+3}) = 2; \quad 0 \le i < p$$

Assign:

$$f(v_n) = 1$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i < t$$
$$f(v_{3i+2}) = 1; \quad 0 \le i < t$$
$$f(v_{3i+3}) = 2; \quad 0 \le i < t$$

Hence f is 3-total super sum cordial labeling.

Case VI: $m \equiv 1\ (mod\ 3)$ and $n \equiv 2\ (mod\ 3)$

Let $m = 3p+1$ and $n = 3t+2$

Assign:

$$f(u_m) = 1$$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p$$
$$f(u_{3i+2}) = 1; \quad 0 \le i < p$$
$$f(u_{3i+3}) = 2; \quad 0 \le i < p$$

Assign:

$$f(v_n) = 1$$
$$f(v_{n-1}) = 2$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i < t$$
$$f(v_{3i+2}) = 1; \quad 0 \le i < t$$
$$f(v_{3i+3}) = 2; \quad 0 \le i < t$$

Hence f is 3-total super sum cordial labeling.

Case VII: $m \equiv 2\ (mod\ 3)$ and $n \equiv 0\ (mod\ 3)$

Let $m = 3p+2$ and $n = 3t$

Assign:

$$f(u_m) = 1$$
$$f(u_{m-1}) = 2$$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i \le p-1$$
$$f(u_{3i+2}) = 1; \quad 0 \le i \le p-1$$
$$f(u_{3i+3}) = 2; \quad 0 \le i \le p-1$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i < t$$
$$f(v_{3i+2}) = 1; \quad 0 \le i < t$$
$$f(v_{3i+3}) = 2; \quad 0 \le i < t$$

Hence f is 3-total super sum cordial labeling.

Case VIII: $m \equiv 2\ (mod\ 3)$ and $n \equiv 1\ (mod\ 3)$

Let $m = 3p+2$ and $n = 3t+1$

Assign

$$f(u_m) = 1$$
$$f(u_{m-1}) = 2$$

Define

$$f(u_{3i+1}) = 2; \quad 0 \le i \le p-1$$
$$f(u_{3i+2}) = 1; \quad 0 \le i \le p-1$$
$$f(u_{3i+3}) = 2; \quad 0 \le i \le p-1$$

Assign

$$f(v_n) = 1$$

Define

$$f(v_{3i+1}) = 2; \quad 0 \le i < t$$
$$f(v_{3i+2}) = 1; \quad 0 \le i < t$$
$$f(v_{3i+3}) = 2; \quad 0 \le i < t$$

Hence f is 3-total super sum cordial labeling.

Case IX: $m \equiv 2\ (mod\ 3)$,$n \equiv 2\ (mod\ 3)$

Let $m = 3p+2$ and $n = 3t+2$

Assign

$$f(u_m) = f(v_n) = 1$$
$$f(u_{m-1}) = f(v_{n-1}) = 2$$

Define:

$$f(u_{3i+1}) = 2; \quad 0 \le i < p$$
$$f(u_{3i+2}) = 1; \quad 0 \le i < p$$
$$f(u_{3i+3}) = 2; \quad 0 \le i < p$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i < t$$
$$f(v_{3i+2}) = 1; \quad 0 \le i < t$$
$$f(v_{3i+3}) = 2; \quad 0 \le i < t$$

Hence f is 3-total super sum cordial labeling.

Table 1: Vertex and edge conditions for 3-Total super sum cordial labeling of $\boldsymbol{p_m \cup p_n}$

Case	Vertex Condition	Edge Condition	$f(i) = v_f(i) + e_f(i)$
m=3p & n=3t	$v_f(0) = 1$ $v_f(1) = 2p + t$ $v_f(2) = p + 2t - 1$	$e_f(0) = 2p + 2t - 2$ $e_f(1) = t$ $e_f(2) = p + 1$	$f(0) = 2p + 2t - 1$ $f(1) = 2p + 2t$ $f(2) = 2p + 2t$
(m=3p & n=3t+1) or (m=3p+1 & n=3t)	$v_f(0) = 0$ $v_f(1) = p + t + 1$ $v_f(2) = 2p + 2t$	$e_f(0) = 2p + 2t$ $e_f(1) = p + t - 1$ $e_f(2) = 0$	$f(0) = 2p + 2t$ $f(1) = 2p + 2t$ $f(2) = 2p + 2t$
(m=3p & n=3t+2)or (m=3p+2 & n=3t)	$v_f(0) = 0$ $v_f(1) = p + t + 1$ $v_f(2) = 2p + 2t + 1$	$e_f(0) = 2p + 2t + 1$ $e_f(1) = p + t - 1$ $e_f(2) = 0$	$f(0) = 2p + 2t + 1$ $f(1) = 2p + 2t$ $f(2) = 2p + 2t + 1$
m=3p+1 & n=3t+1	$v_f(0) = 0$ $v_f(1) = p + t + 1$ $v_f(2) = 2p + 2t + 1$	$e_f(0) = 2p + 2t + 1$ $e_f(1) = p + t - 1$ $e_f(2) = 0$	$f(0) = 2p + 2t + 1$ $f(1) = 2p + 2t$ $f(2) = 2p + 2t + 1$
(m=3p+1 & n=3t+2)or (m=3p+2 & n=3t+1)	$v_f(0) = 0$ $v_f(1) = p + t + 2$ $v_f(2) = 2p + 2t + 1$	$e_f(0) = 2p + 2t + 2$ $e_f(1) = p + t - 1$ $e_f(2) = 0$	$f(0) = 2p + 2t + 2$ $f(1) = 2p + 2t + 1$ $f(2) = 2p + 2t + 1$
m=3p+2 & n=3t+2	$v_f(0) = 0$ $v_f(1) = p + t + 2$ $v_f(2) = 2p + 2t + 2$	$e_f(0) = 2p + 2t + 2$ $e_f(1) = p + t$ $e_f(2) = 0$	$f(0) = 2p + 2t + 2$ $f(1) = 2p + 2t + 2$ $f(2) = 2p + 2t + 2$

Example 2.2.: A 3-Total super sum cordial labeling of $P_6 \cup P_7$

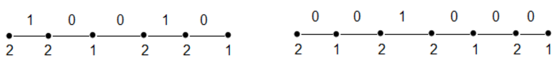

Figure 1 : $\mathbf{P_6 \cup P_7}$

Theorem 2.3.: $k_1, m \cup k_1, n$ is 3-total super sum cordial.

Proof: Let $V(k_1, m) = \{u, u_i : 1 \le i \le m\}$ and $E(k_1, m) = \{uu_i : 1 \le i \le m\}$ and $V(k_1, n) = \{v, v_i : 1 \le i \le n\}$ and $E(k_1, n) = \{vv_i : 1 \le i \le n\}$

Case I: $m \equiv 0\ (mod\ 3), n \equiv 0\ (mod\ 3)$

Let $m = 3p$ and $n = 3t$

Assign

$$f(u) = 1$$

Define

$$f(u_{3i+1}) = 2; \quad 0 \le i < p$$
$$f(u_{3i+2}) = 0; \quad 0 \le i < p$$
$$f(u_{3i+3}) = 1; \quad 0 \le i < p$$

Assign

$$f(v) = 1; \quad f(v_{n-2}) = 1$$
$$f(v_n) = f(v_{n-1}) = 2$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i < t - 1$$
$$f(v_{3i+2}) = 0; \quad 0 \le i < t - 1$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t - 1$$

Hence f is 3-total super sum cordial labeling.

Case II: $m \equiv 0\ (mod\ 3), n \equiv 1\ (mod\ 3)$

Let $m = 3p$ and $n = 3t + 1$

Assign

$$f(u) = 1$$

Define

$$f(u_{3i+1}) = 2; \quad 0 \le i < p$$
$$f(u_{3i+2}) = 0; \quad 0 \le i < p$$
$$f(u_{3i+3}) = 1; \quad 0 \le i < p$$

Assign

$$f(v) = 1$$
$$f(v_n) = 2$$

Define

$$f(v_{3i+1}) = 2; \quad 0 \le i < t$$
$$f(v_{3i+2}) = 0; \quad 0 \le i < t$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t$$

Hence f is 3-total super sum cordial labeling.

Case III: $m \equiv 0\ (mod\ 3), n \equiv 2\ (mod\ 3)$

Let $m = 3p$ and $n = 3t + 2$

Assign

$$f(u) = 1$$

Define

$$f(u_{3i+1}) = 2; \quad 0 \le i < p$$
$$f(u_{3i+2}) = 0; \quad 0 \le i < p$$
$$f(u_{3i+3}) = 1; \quad 0 \le i < p$$

Assign

$$f(v) = 1$$
$$f(v_n) = f(v_{n-1}) = 2$$

Define:

$$f(v_{3i+1}) = 2; \quad 0 \le i < t$$
$$f(v_{3i+2}) = 0; \quad 0 \le i < t$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t$$

Hence f is 3-total super sum cordial labeling.

Case IV: $m \equiv 1\ (mod\ 3), n \equiv 0\ (mod\ 3)$

Label k_1, m as k_1, n is labeled in case II and label k_1, n as k_1, m is labeled in case II.

Hence f is 3-total super sum cordial labeling.

Case V: $m \equiv 1\ (mod\ 3), n \equiv 1\ (mod\ 3)$

Let $m = 3p + 1$ and $n = 3t + 1$

Assign

$$f(u) = 1$$
$$f(u_m) = 2$$

International Journal of Applied Information Systems (IJAIS) – ISSN : 2249-0868
Foundation of Computer Science FCS, New York, USA
Volume 8– No.4, February 2015 – www.ijais.org

Define

$$f(u_{3i+1}) = 2; \quad 0 \le i < p$$
$$f(u_{3i+2}) = 0; \quad 0 \le i < p$$
$$f(u_{3i+3}) = 1; \quad 0 \le i < p$$

Assign

$$f(v) = 1$$
$$f(v_n) = 2$$

Define

$$f(v_{3i+1}) = 2; \quad 0 \le i < t$$
$$f(v_{3i+2}) = 0; \quad 0 \le i < t$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t$$

Hence f is 3-total super sum cordial labeling.

Case VI: $m \equiv 1 \ (mod\ 3), n \equiv 2 \ (mod\ 3)$

Let $m = 3p + 1$ and $n = 3t + 2$

Assign

$$f(u) = 1$$
$$f(u_m) = 2$$

Define

$$f(u_{3i+1}) = 2; \quad 0 \le i < p$$
$$f(u_{3i+2}) = 0; \quad 0 \le i < p$$
$$f(u_{3i+3}) = 1; \quad 0 \le i < p$$

Assign

$$f(v) = 1$$
$$f(v_n) = 2$$
$$f(v_{n-1}) = 0$$

Define

$$f(v_{3i+1}) = 2; \quad 0 \le i < t$$
$$f(v_{3i+2}) = 0; \quad 0 \le i < t$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t$$

Hence f is 3-total super sum cordial labeling.

Case VII: $m \equiv 2 \ (mod\ 3), n \equiv 0 \ (mod\ 3)$

Label k_1, m as k_1, n is labeled in case III and label k_1, n as k_1, m is labeled in case III.

Hence f is 3-total super sum cordial labeling.

Case VIII: $m \equiv 2 \ (mod\ 3), n \equiv 1 \ (mod\ 3)$

Label k_1, m as k_1, n is labeled in case VI and label k_1, n as k_1, m is labeled in case VI.

Hence f is 3-total super sum cordial labeling.

Case IX: $m \equiv 2 \ (mod\ 3), n \equiv 2 \ (mod\ 3)$

Let $m = 3p + 2$ and $n = 3t + 2$

Assign

$$f(u) = 1$$
$$f(u_m) = 2$$
$$f(u_{m-1}) = 0$$

Define

$$f(u_{3i+1}) = 2; \quad 0 \le i < p$$
$$f(u_{3i+2}) = 0; \quad 0 \le i < p$$
$$f(u_{3i+3}) = 1; \quad 0 \le i < p$$

Assign

$$f(v) = 1$$
$$f(v_n) = 2$$
$$f(v_{n-1}) = 1$$

Define

$$f(v_{3i+1}) = 2; \quad 0 \le i < t$$
$$f(v_{3i+2}) = 0; \quad 0 \le i < t$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t$$

Hence f is 3-total super sum cordial labeling.

Table 2: Vertex and edge conditions for 3-Total super sum cordial labeling of $k_1, m \cup k_1, n$

Case	Vertex Condition	Edge Condition	$f(i) = v_f(i) + e_f(i)$
m=3p & n=3t	$v_f(0) = p + t - 1$ $v_f(1) = p + t + 2$ $v_f(2) = p + t + 1$	$e_f(0) = p + t + 1$ $e_f(1) = p + t - 1$ $e_f(2) = p + t$	$f(0) = 2p + 2t$ $f(1) = 2p + 2t + 1$ $f(2) = 2p + 2t + 1$
(m=3p & n=3t+1) or (m=3p+1 & n=3t)	$v_f(0) = p + t$ $v_f(1) = p + t + 2$ $v_f(2) = p + t + 1$	$e_f(0) = p + t + 1$ $e_f(1) = p + t$ $e_f(2) = p + t$	$f(0) = 2p + 2t + 1$ $f(1) = 2p + 2t + 2$ $f(2) = 2p + 2t + 1$
(m=3p & n=3t+2) or (m=3p+2 & n=3t)	$v_f(0) = p + t$ $v_f(1) = p + t + 2$ $v_f(2) = p + t + 2$	$e_f(0) = p + t + 2$ $e_f(1) = p + t$ $e_f(2) = p + t$	$f(0) = 2p + 2t + 2$ $f(1) = 2p + 2t + 2$ $f(2) = 2p + 2t + 2$
m=3p+1 & n=3t+1	$v_f(0) = p + t$ $v_f(1) = p + t + 2$ $v_f(2) = p + t + 2$	$e_f(0) = p + t + 2$ $e_f(1) = p + t$ $e_f(2) = p + t$	$f(0) = 2p + 2t + 2$ $f(1) = 2p + 2t + 2$ $f(2) = 2p + 2t + 2$
(m=3p+1 & n=3t+2) or (m=3p+2 & n=3t+1)	$v_f(0) = p + t + 1$ $v_f(1) = p + t + 2$ $v_f(2) = p + t + 2$	$e_f(0) = p + t + 2$ $e_f(1) = p + t + 1$ $e_f(2) = p + t$	$f(0) = 2p + 2t + 3$ $f(1) = 2p + 2t + 3$ $f(2) = 2p + 2t + 2$
m=3p+2 & n=3t+2	$v_f(0) = p + t + 1$ $v_f(1) = p + t + 3$ $v_f(2) = p + t + 2$	$e_f(0) = p + t + 2$ $e_f(1) = p + t + 1$ $e_f(2) = p + t + 1$	$f(0) = 2p + 2t + 3$ $f(1) = 2p + 2t + 4$ $f(2) = 2p + 2t + 3$

Example 2.4.: A 3-total super sum cordial labeling of $k_1, 5 \cup k_1, 9$.

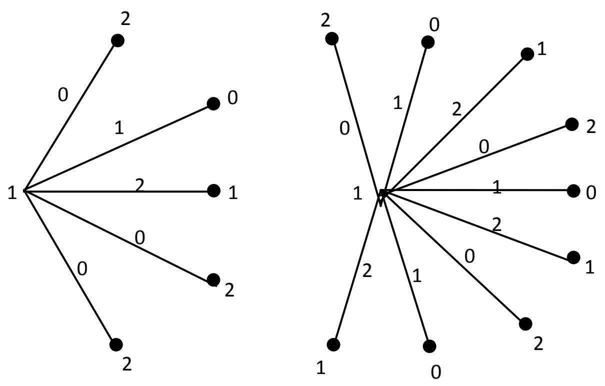

Figure 2 : $\mathbf{k_1, 5 \cup k_1, 9}$

Theorem 2.5.: $c_m \cup c_n$ is 3-total super sum cordial.

Proof: Let c_m be the cycle $u_1, u_2, \dots, u_m, u_1$ and c_n be the cycle $v_1, v_2, \dots, v_n, v_1$

Case I: $m \equiv 0 \ (mod\ 3), n \equiv 0 \ (mod\ 3)$

Let $m = 3p$ and $n = 3t$

Define

$$f(u_{3i+1}) = 2; \quad 0 \le i < p$$

$$f(u_{3i+2}) = 2; \quad 0 \le i < p$$
$$f(u_{3i+3}) = 1; \quad 0 \le i < p$$
$$f(v_{3i+1}) = 2; \quad 0 \le i < t$$
$$f(v_{3i+2}) = 2; \quad 0 \le i < t$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t$$

Hence f is 3-total super sum cordial labeling.

Case II: $m \equiv 0\ (mod\ 3)$, $n \equiv 1\ (mod\ 3)$

Let $m = 3p$ and $n = 3t + 1$

Define

$$f(u_{3i+1}) = 2; \quad 0 \le i < p$$
$$f(u_{3i+2}) = 2; \quad 0 \le i < p$$
$$f(u_{3i+3}) = 1; \quad 0 \le i < p$$

Assign

$$f(v_n) = 1$$

Define

$$f(v_{3i+1}) = 2; \quad 0 \le i < t$$
$$f(v_{3i+2}) = 2; \quad 0 \le i < t$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t$$

Hence f is 3-total super sum cordial labeling.

Case III: $m \equiv 0\ (mod\ 3)$, $n \equiv 2\ (mod\ 3)$

Let $m = 3p$ and $n = 3t + 2$

Define

$$f(u_{3i+1}) = 2; \quad 0 \le i < p$$
$$f(u_{3i+2}) = 2; \quad 0 \le i < p$$
$$f(u_{3i+3}) = 1; \quad 0 \le i < p$$

Assign

$$f(v_n) = 1$$
$$f(v_{n-1}) = 2$$

Define

$$f(v_{3i+1}) = 2; \quad 0 \le i < t$$
$$f(v_{3i+2}) = 1; \quad 0 \le i < t$$
$$f(v_{3i+3}) = 2; \quad 0 \le i < t$$

Hence f is 3-total super sum cordial labeling.

Case IV: $m \equiv 1\ (mod\ 3)$ and $n \equiv 0\ (mod\ 3)$

Let $m = 3p + 1$ and $n = 3t$

Label c_m as c_n is labeled in case II and Label c_n as c_m is labeled in case II

Hence f is 3-total super sum cordial labeling.

Case V: $m \equiv 1\ (mod\ 3)$ and $n \equiv 1\ (mod\ 3)$

Let $m = 3p + 1$ and $n = 3t + 1$

Assign

$$f(u_m) = 1$$

Define

$$f(u_{3i+1}) = 2; \quad 0 \le i < p$$
$$f(u_{3i+2}) = 2; \quad 0 \le i < p$$
$$f(u_{3i+3}) = 1; \quad 0 \le i < p$$

Assign

$$f(v_n) = 1$$

Define

$$f(v_{3i+1}) = 2; \quad 0 \le i < t$$
$$f(v_{3i+2}) = 1; \quad 0 \le i < t$$
$$f(v_{3i+3}) = 2; \quad 0 \le i < t$$

Hence f is 3-total super sum cordial labeling.

Case VI: $m \equiv 1\ (mod\ 3)$ and $n \equiv 2\ (mod\ 3)$

Let $m = 3p + 1$ and $n = 3t + 2$

Assign

$$f(u_m) = 1$$

Define

$$f(u_{3i+1}) = 2; \quad 0 \le i < p$$
$$f(u_{3i+2}) = 2; \quad 0 \le i < p$$
$$f(u_{3i+3}) = 1; \quad 0 \le i < p$$

Assign

$$f(v_n) = 1$$
$$f(v_{n-1}) = 2$$

Define

$$f(v_{3i+1}) = 2; \quad 0 \le i < t$$
$$f(v_{3i+2}) = 1; \quad 0 \le i < t$$
$$f(v_{3i+3}) = 2; \quad 0 \le i < t$$

Hence f is 3-total super sum cordial labeling.

Case VII: $m \equiv 2\ (mod\ 3)$, $n \equiv 0\ (mod\ 3)$

Let $m = 3p + 2$ and $n = 3t$

Label c_m as c_n is labeled in case III and Label c_n as c_m is labeled in case III

Hence f is 3-total super sum cordial labeling.

Case VIII: $m \equiv 2\ (mod\ 3)$, $n \equiv 1\ (mod\ 3)$

Let $m = 3p + 2$ and $n = 3t + 1$

Label c_m as c_n is labeled in case VI and Label c_n as c_m is labeled in case VI

Hence f is 3-total super sum cordial labeling.

Case IX: $m \equiv 2\ (mod\ 3)$, $n \equiv 2\ (mod\ 3)$

Let $m = 3p + 2$ and $n = 3t + 2$

Assign

$$f(u_m) = 1$$
$$f(u_{m-1}) = 2$$

Define

$$f(u_{3i+1}) = 2; \quad 0 \le i < p$$
$$f(u_{3i+2}) = 1; \quad 0 \le i < p$$
$$f(u_{3i+3}) = 2; \quad 0 \le i < p$$

Assign

$$f(v_n) = 2$$
$$f(v_{n-1}) = 2$$

Define

$$f(v_{3i+1}) = 2; \quad 0 \le i < t$$
$$f(v_{3i+2}) = 2; \quad 0 \le i < t$$
$$f(v_{3i+3}) = 1; \quad 0 \le i < t$$

Hence f is 3-total super sum cordial labeling.

Table 3: Vertex and edge conditions for 3-Total super sum cordial labeling of $c_m \cup c_n$

Case	Vertex Condition	Edge Condition	$f(i) = v_f(i) + e_f(i)$
m=3p & n=3t	$v_f(0) = 0$ $v_f(1) = p + t$ $v_f(2) = 2p + 2t$	$e_f(0) = 2p + 2t$ $e_f(1) = p + t$ $e_f(2) = 0$	$f(0) = 2p + 2t$ $f(1) = 2p + 2t$ $f(2) = 2p + 2t$
(m=3p & n=3t+1) or (m=3p+1 & n=3t)	$v_f(0) = 0$ $v_f(1) = p + t + 1$ $v_f(2) = 2p + 2t$	$e_f(0) = 2p + 2t$ $e_f(1) = p + t$ $e_f(2) = 1$	$f(0) = 2p + 2t$ $f(1) = 2p + 2t + 1$ $f(2) = 2p + 2t + 1$
(m=3p & n=3t+2)or (m=3p+2 & n=3t)	$v_f(0) = 0$ $v_f(1) = p + t + 1$ $v_f(2) = 2p + 2t + 1$	$e_f(0) = 2p + 2t + 2$ $e_f(1) = p + t$ $e_f(2) = 0$	$f(0) = 2p + 2t + 2$ $f(1) = 2p + 2t + 1$ $f(2) = 2p + 2t + 1$
m=3p+1 & n=3t+1	$v_f(0) = 0$ $v_f(1) = p + t + 2$ $v_f(2) = 2p + 2t$	$e_f(0) = 2p + 2t + 2$ $e_f(1) = p + t - 1$ $e_f(2) = 1$	$f(0) = 2p + 2t + 2$ $f(1) = 2p + 2t + 1$ $f(2) = 2p + 2t + 1$
(m=3p+1 & n=3t+2)or (m=3p+2 & n=3t+1)	$v_f(0) = 0$ $v_f(1) = p + t + 2$ $v_f(2) = 2p + 2t + 1$	$e_f(0) = 2p + 2t + 2$ $e_f(1) = p + t$ $e_f(2) = 1$	$f(0) = 2p + 2t + 2$ $f(1) = 2p + 2t + 2$ $f(2) = 2p + 2t + 2$
m=3p+2 & n=3t+2	$v_f(0) = 0$ $v_f(1) = p + t + 1$ $v_f(2) = 2p + 2t + 3$	$e_f(0) = 2p + 2t + 2$ $e_f(1) = p + t + 2$ $e_f(2) = 0$	$f(0) = 2p + 2t + 2$ $f(1) = 2p + 2t + 3$ $f(2) = 2p + 2t + 3$

Example 2.6.: A 3-total super sum cordial labeling $c_7 \cup c_5$

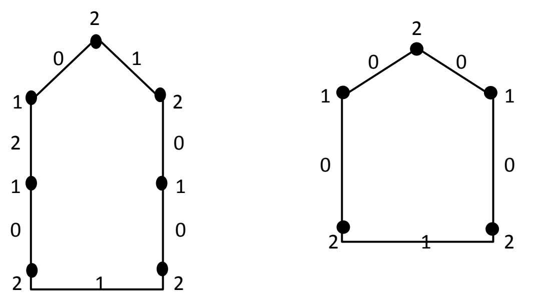

Figure 3 :$C_7 \cup C_5$

3. CONCLUSION

If $G_1 \cup G_2$ is 2-total sum cordial graph then it is 2-total super sum cordial graph, as for each edge uv $|f(u) - f(v)| \le 1$.

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