



On Intuitionistic Fuzzy Entropy as Cost Function in Image Denoising

Rajeev Kaushik
Research Scholar,
Singhania University,
Pacheri Bari, Rajasthan, India

Rakesh K Bajaj,
Associate Professor, Department of Mathematics,
JUIT, Wagnaghat,
Solan, Himachal Pradesh, India

ABSTRACT

In this paper, we proposed an algorithm to find the optimal threshold value for denoising an image. A new cost function is designed to find the optimal threshold in every image. The cost function is based on the intuitionistic fuzzy divergence measure of the denoised image and original image. In addition, the intuitionistic fuzzy entropy of denoised image is added to the cost function. This is necessary, because when the algorithm threshold value is decreased, the denoised image is blurred, although its divergence of original image is decreased. When the value of intuitionistic fuzzy entropy and intuitionistic fuzzy divergence measure are minimum, the sum that is the cost value will also minimum. The threshold for image denoising with a minimum cost value will be the optimal threshold for image denoising. The implementation and applicability of the proposed algorithm have been illustrated by taking different sample images. The obtained results have been finally analyzed and found to be better than the existing ones.

Keywords:

Intuitionistic Fuzzy sets, Intuitionistic Fuzzy Entropy, Intuitionistic Divergence measure, Intuitionistic Cost function, Image denoising.

1. INTRODUCTION

To deal with fuzziness, Zadeh [1] introduced the concept of fuzzy set and later on, different notions of higher-order FSs have been presented by various researchers. Among them, intuitionistic fuzzy sets (IFSs) proposed by Atanassov [2] - [6], turned out to be a suitable tool for modelling the hesitancy arising from imprecise or/and imperfect information. IFSs are defined using two characteristic functions, namely the membership and the non-membership, describing the belongingness or non-belongingness of an element of the universe to the IFS respectively. Entropy of a fuzzy set describes the fuzziness degree of a fuzzy set and was first mentioned by Zadeh [7] in 1968. A lot of interest in research community generated to study it from different points of view. Yager [17] was to view the fuzziness degree of a fuzzy set in terms of a lack of distinction between the fuzzy set and its complement. In 1972, Deluca and Termini [8] presented some axioms to describe the fuzziness degree of fuzzy set, and also proposed fuzzy entropy based on Shannon's [12] function. Later on in 1975, Kaufmann [9] proposed a method for measuring the fuzziness degree of a fuzzy set by a metric distance between its membership function and the membership function of its nearest crisp set. Considering the degree of hesitancy, Atanassov(1986) defined intuitionistic fuzzy sets as the generalization of fuzzy sets. Bustince and Burillo [10] firstly introduced an entropy on IFS in 1996, and then Hung [18], Zhang *et al.* [19], Vlachos and Sergiadis [20] presented different entropies on IFS from different aspects. Szmidt and Kacprzyk [14],[15] introduced the entropy for intuitionistic fuzzy sets and its axioms.

Divergence measures based on the idea of information-theoretic entropy were first introduced in communication theory by Shan-

non [12] and later by Wiener [13] in Cybernetics. The most popular divergence measure associated with the Shannon entropy function is the Kullback-Leibler divergence (K-L divergence) [11], perhaps because of its simplicity. It has been indicated that the use of different entropy measures may lead to different models than those obtained by Shannon and Kullback - Leibler measures.

Impulse noise in images is present due to bit errors in transmission or introduced during the signal acquisition stage. There are two types of impulse noise, they are salt and pepper noise and random valued noise. Salt and pepper noise can corrupt the images where the corrupted pixel takes either maximum or minimum gray level. Several nonlinear filters have been proposed for restoration of images contaminated by salt and pepper noise. Among these standard median filter has been established as reliable method to remove the salt and pepper noise without damaging the edge details. However, the major drawback of standard Median Filter (MF) is that the filter is effective only at low noise densities [23]. When the noise level is over 50 percent the edge details of the original image will not be preserved by standard median filter. Adaptive Median Filter (AMF) [24] perform well at low noise densities. But at high noise densities the window size has to be increased which may lead to blurring the image. In switching median filter [25], [26] the decision is based on a pre-defined threshold value. The major drawback of this method is that defining a robust decision is difficult. Also these filters will not take into account the local features as a result of which details and edges may not be recovered satisfactorily, especially when the noise level is high. To overcome the above drawback, Decision Based Algorithm (DBA) is proposed [27]. In this, image is denoised by using a 3×3 window. If the processing pixel value is 0 or 255 it is processed or else it is left unchanged. At high noise density the median value will be 0 or 255 which is noisy. In such case, neighboring pixel is used for replacement. This repeated replacement of neighboring pixel produces streaking effect [28].

2. PRELIMINARIES

Atanassov's intuitionistic fuzzy set (IFS) over a finite non empty fixed set X , is a set \tilde{A} defined as

$\tilde{A} = \{ \langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) \rangle \mid x \in X \}$ which assigns to each element $x \in X$ to the set \tilde{A} , which is subset of X having the degree of membership $\mu_{\tilde{A}}(x) : X \rightarrow [0, 1]$ and degree of non-membership $\nu_{\tilde{A}}(x) : X \rightarrow [0, 1]$, satisfying $0 \leq \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1$, for all $x \in X$. We denote $\mathcal{IFS}(X)$ the set of all the IFSs on X .

For each intuitionistic fuzzy set in X , a hesitation margin $\pi_{\tilde{A}}(x)$, whether x belongs to \tilde{A} or not, which is the intuitionistic fuzzy index of element x in the IFS \tilde{A} , defined by $\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x)$, denotes a measure of non-determinacy. It may be noted that the application of intuitionistic fuzzy sets instead of fuzzy sets introduces another degree of freedom into a set description (i.e. in addition to $\mu_{\tilde{A}}$, we also have $\nu_{\tilde{A}}$ or $\pi_{\tilde{A}}$.



De Luca and Termini(1972) proposed non-probabilistic entropy for FSs and formulated the axiomatic requirement with which an entropy measure should comply, Szmidt and Kacprzyk(2001) extended the axioms of De Luca and Termini(1972), proposing the following definition for an entropy measure in the setting of IFSs theory.

DEFINITION 1. (Szmidt and Kacprzyk, 2001) An entropy on IFS(X) is a real-valued functional $E : \mathcal{IFS}(X) \rightarrow [0, 1]$, satisfies the following axiomatic requirements:

- (1) $E(\tilde{A}) = 0$ if and only if A is a crisp set, that is $\mu_{\tilde{A}(x_i)=0}$ or $\mu_{\tilde{A}(x_i)=1}$ for all $x_i \in X$
- (2) $E(\tilde{A}) = 1$ if and only if $\mu_{\tilde{A}(x_i)} = \nu_{\tilde{A}(x_i)}$ for all $x_i \in X$, that is $\tilde{A} = \tilde{A}^c$
- (3) $E(\tilde{A}) \leq E(\tilde{B})$ if \tilde{A} is sharper than \tilde{B} , i.e. $\mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x)$ and $\nu_{\tilde{A}}(x) \geq \nu_{\tilde{B}}(x)$, for $\mu_{\tilde{B}}(x) \leq \nu_{\tilde{B}}(x)$ or $\mu_{\tilde{A}}(x) \geq \mu_{\tilde{B}}(x)$ and $\nu_{\tilde{A}}(x) \leq \nu_{\tilde{B}}(x)$, for $\mu_{\tilde{B}}(x) \geq \nu_{\tilde{B}}(x)$ for all $x_i \in X$
- (4) $E(\tilde{A}) = E(\tilde{A}^c)$

Moreover, an intuitionistic fuzzy entropy measure was defined as:

$$E(A) = \frac{1}{n} \sum_{i=1}^n \frac{\max \text{Count}(A_i \cap A_i^c)}{\max \text{Count}(A_i \cup A_i^c)} \quad (1)$$

where, n is cardinal (X) and A_i denotes the single-element IFS corresponding to the i -th element of the universe X . In other words, A_i is the i th component of A . Moreover, $\max \text{Count}(A)$ denotes the biggest cardinality of A and is given by

$$\max \text{Count}(A) = \sum_{i=1}^n n(\mu_A(x_i) + \pi_A(x_i)) \quad (2)$$

Zeng et al.(2008) introduced the concept of similarity measure of intuitionistic fuzzy sets and proposed a new method for describing entropy of intuitionistic fuzzy set based on similarity measure of intuitionistic fuzzy sets. These also introduced some formulas to calculate entropy of intuitionistic fuzzy sets.

DEFINITION 2. Simple intuitionistic fuzzy entropy for IFS A can be defined as:

$$E(A) = \frac{1}{n} \sum_{i=1}^n e(x_i) \quad (3)$$

where

$$e(x_i) = 1 - \sqrt{(\mu_A(x_i) - \nu_A(x_i))^2}$$

It satisfies Szmidt and Kacprzyk, (2001).

DEFINITION 3. (Burillo and Bustince) Let $X = \{x_1, x_2, x_3, \dots, x_N\}$, $I : \mathcal{IFSs}(X) \rightarrow R^+$, and $A \in \mathcal{IFS}(X)$.

If $I(A) = \sum_{i=1}^N (1 - \phi(\mu_A(x_i), \nu_A(x_i)))$, where ϕ verify the condition given below, then I is an intuitionistic entropy.

- (1) $I(A) = 0$ if and only if $\mu_A(x_i) + \nu_A(x_i) = 1$ for all x_i .
- (2) $I(A) = N$ if and only if $\mu_A(x_i) = \nu_A(x_i) = 0$ for all x_i .
- (3) $I(A) = I(A^c)$,
- (4) If $\mu_A(x_i) \leq \mu_B(x_i)$ and $\nu_A(x_i) \geq \nu_B(x_i)$ i.e. $A \leq B$ then $I(A) \geq I(B)$.

3. EXPONENTIAL ENTROPY OF ORDER β OF INTUITIONISTIC FUZZY SET

Here the axiomatic definition for the entropy of the intuitionistic fuzzy sets is presented. The following conditions give the intuitionistic idea for the degree of fuzziness of the intuitionistic fuzzy set, i.e., for the entropy of the intuitionistic fuzzy set:

- (1) It will be null when the intuitionistic fuzzy set is a fuzzy set;
- (2) It will be maximum if the intuitionistic fuzzy set is completely intuitionistic;
- (3) An intuitionistic entropy of the intuitionistic fuzzy set will be equal to its complement;
- (4) If the degree of membership and the degree of non-membership of each element increase, the sum will do so as well, and therefore, this intuitionistic fuzzy set becomes less fuzzy, and therefore the entropy should decrease.

In view of the above stated points and the axiomatic definition of entropy for an intuitionistic fuzzy set is given as

DEFINITION 4. A real-valued function $E : \mathcal{IFS}(X) \rightarrow [0, 1]$ is called the entropy measure on $\mathcal{IFS}(X)$, if E satisfies the following properties:

- (E1.) $E(\tilde{A}) = 0 \Leftrightarrow \tilde{A}$ is a fuzzy set;
- (E2.) $E(\tilde{A}) = 1 \Leftrightarrow \mu_{\tilde{A}}(x) = \nu_{\tilde{A}}(x), \forall x \in X$;
- (E3.) $E(\tilde{A}) \leq E(\tilde{B})$ if \tilde{A} is less fuzzy than \tilde{B} , i.e., $\mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x)$ and $\nu_{\tilde{A}}(x) \geq \nu_{\tilde{B}}(x)$ for $\mu_{\tilde{B}}(x) \leq \nu_{\tilde{B}}(x)$ or $\mu_{\tilde{A}}(x) \geq \mu_{\tilde{B}}(x)$ and $\nu_{\tilde{A}}(x) \leq \nu_{\tilde{B}}(x)$ for $\mu_{\tilde{B}}(x) \geq \nu_{\tilde{B}}(x), \forall x \in X$;
- (E4.) $E(\tilde{A}) = E(\tilde{A}^c)$, where \tilde{A}^c is the complement of \tilde{A} .

In paper title [29] we proposed the intuitionistic fuzzy entropy of order β which is defined as

$$E^\beta(\tilde{A}) = \frac{1}{n} \sum_{i=1}^n \left[1 - (\mu(x_i) + \nu(x_i))^\beta \cdot e^{1 - (\mu(x_i) + \nu(x_i))^\beta} \right], \quad (4)$$

where $\beta \in [0, 1]$. An entropy (4) is a valid intuitionistic fuzzy entropy of intuitionistic fuzzy set.

3.1 Intuitionistic Fuzzy Divergence Measure

Divergence measures based on entropy functions present the dissimilarity between pairs of probability distributions, and are therefore widely used for the process of statistical inference. Divergence measures between intuitionistic fuzzy sets play a significant role in many applications of intuitionistic fuzzy set theory viz. image processing, pattern recognition and decision making problems etc. The divergence measure between IFSs \tilde{A} and \tilde{B} is denoted by $D_{IF}^\beta(\tilde{A}, \tilde{B})$ and satisfies the following properties (D1-D3):

- (D1.) $0 \leq D_{IF}^\beta(\tilde{A}, \tilde{B}) \leq 1$ and as $D_{IF}^\beta(\tilde{A}, \tilde{B}) = 0$, if and only if $\tilde{A} = \tilde{B}$;
- (D2.) $D_{IF}^\beta(\tilde{A}, \tilde{B}) = D_{IF}^\beta(\tilde{B}, \tilde{A})$;
- (D3.) Let \tilde{A}, \tilde{B} and \tilde{C} be IFSs in X , such that $\tilde{A} \subseteq \tilde{B} \subseteq \tilde{C}$. Then $D_{IF}^\beta(\tilde{A}, \tilde{B}) \leq D_{IF}^\beta(\tilde{A}, \tilde{C})$ and $D_{IF}^\beta(\tilde{B}, \tilde{C}) \leq D_{IF}^\beta(\tilde{A}, \tilde{C})$.

Let $\tilde{A} = \{(x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) | x \in X)\}$ and $\tilde{B} = \{(x, \mu_{\tilde{B}}(x), \nu_{\tilde{B}}(x) | x \in X)\}$ be two intuitionistic fuzzy sets over the single point universal set $X = \{x\}$. Considering the hesitation degree, the interval or range of the membership degree of the two intuitionistic fuzzy sets \tilde{A} and \tilde{B} may be represented as $[\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(x) + \pi_{\tilde{A}}(x)]$ and $[\mu_{\tilde{B}}(x), \mu_{\tilde{B}}(x) + \pi_{\tilde{B}}(x)]$, respectively. The interval is due to the hesitation or the lack of knowledge in assigning membership values.

In paper title [29] we proposed the intuitionistic divergence measure between two IFSs \tilde{A} and \tilde{B} based on the above entropy (4) which is defined as:

$$D_{IF}^\beta(\tilde{A}, \tilde{B}) = \left(\frac{E^\beta(\tilde{A}) + E^\beta(\tilde{B})}{2} \right) - E^\beta \left(\frac{\tilde{A} + \tilde{B}}{2} \right)$$



$$D_{IF}^{\beta}(\tilde{A}, \tilde{B}) = \left[\frac{1}{2} \left[1 - (\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x))^{\beta} \right] \cdot e^{1 - (\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x))^{\beta}} \right. \\ \left. + \frac{1}{2} \left[1 - (\mu_{\tilde{B}}(x) + \nu_{\tilde{B}}(x))^{\beta} \right] \cdot e^{1 - (\mu_{\tilde{B}}(x) + \nu_{\tilde{B}}(x))^{\beta}} \right] \\ \left[1 - \left(\frac{\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x) + \nu_{\tilde{B}}(x)}{2} \right)^{\beta} \right] \\ \cdot e^{1 - \left(\frac{\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x) + \nu_{\tilde{B}}(x)}{2} \right)^{\beta}} \right] \quad (5)$$

The Intuitionistic Fuzzy Divergence Measure (5) is a valid Divergence Measure.

4. IMAGE DENOISING USING PROPOSED INTUITIONISTIC FUZZY COST FUNCTION

A few fuzzy image denoising methods have been found in literature. E.Pasha *et al.* [21] applied fuzzy entropy as cost function in denoising the image and find out that fuzzy entropy as a measure of image blurring has an important role in image processing. A. Fatami [22] introduced entropy, cross-entropy and discrimination measure for stochastic intuitionistic fuzzy sets, where cost function was introduced as the summation of the distance between original image, denoised image and fuzzy entropy of denoised image because in addition to the distance of the original image and the denoised image, the entropy of the denoised image should be considered as a blurring caused by the replacement of noised pixels. In this section, we designed the cost function based on proposed intuitionistic fuzzy divergence measure, intuitionistic fuzzy entropy [29] and propose a denoising algorithm using median filtering with predefined threshold.

Let $\tilde{A} = \{(x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)) | x \in X\}$ and $\tilde{B} = \{(x, \mu_{\tilde{B}}(x), \nu_{\tilde{B}}(x)) | x \in X\}$ be two intuitionistic fuzzy sets over the single point universal set $X = \{x\}$. Considering the hesitation degree, the interval or range of the membership degree of the two intuitionistic fuzzy sets \tilde{A} and \tilde{B} may be represented as $[\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(x) + \pi_{\tilde{A}}(x)]$ and $[\mu_{\tilde{B}}(x), \mu_{\tilde{B}}(x) + \pi_{\tilde{B}}(x)]$, respectively. The interval is due to the hesitation or the lack of knowledge in assigning membership values. For calculating the intuitionistic fuzzy index i.e. to know the hesitation degree, taken as

$$\text{Intuitionistic fuzzy index} = c * (1 - \text{membership value}) \quad (6)$$

Where c is the hesitation constant.

4.1 Intuitionistic Fuzzy Cost Function

Let \tilde{A} be a intuitionistic fuzzy version of original image and \tilde{C} be the denoised image, intuitionistic fuzzy entropy is calculated as the sum of distance between \tilde{A} , \tilde{C} and intuitionistic fuzzy entropy of \tilde{C} , because in addition to distance of original image and denoised image, it should be considered that intuitionistic entropy of denoised image as blurring due to change of noised pixels with medium of 8 neighbor pixels. The cost function is given as under:

$$C(\tilde{A}) = E^{\beta}(\tilde{A}) + D_{IF}^{\beta}(\tilde{A}, \tilde{B}), \quad (7)$$

where $C(\tilde{A})$ is cost function, $E^{\beta}(\tilde{A})$ is entropy of intuitionistic fuzzy set and $D_{IF}^{\beta}(\tilde{A}, \tilde{B})$ is intuitionistic fuzzy divergence measure. The adaptive median filtering with a predefined threshold has been applied in the proposed algorithm. The median filtering performs spatial processing to determine which pixels in an image have been affected by noise. The median filter classifies pixels as noise by comparing each pixel in the image to its

surrounding neighbor pixels. The size of the neighborhood is adjustable, as well as the threshold for the comparison. A pixel that is different from a majority of its neighbors, as well as being not structurally aligned with those pixels to which it is similar, is labeled as noise pixel. These noise pixels are then replaced by the median pixel value of the pixels in the neighborhood that have passed the noise labeling test.

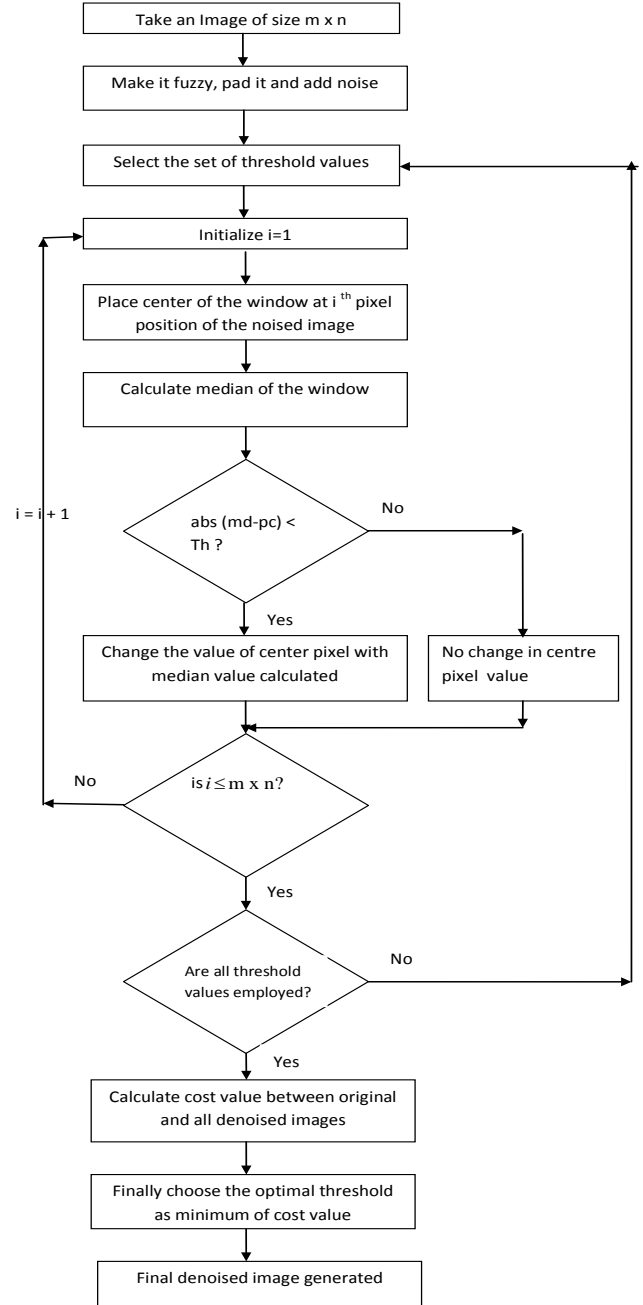


Fig. 1. Flow Chart

4.2 Proposed Algorithm

- Step 1. Select an original image.
- Step 2. Make it fuzzy image, applying padding and add noise into it.
- Step 3. Select the set of threshold values.



- Step 4. For each threshold value do the steps from 5-8.
- Step 5. Place the center of the window of size 3×3 at $(i, j)^{th}$ pixel position of the noised image.
- Step 6. Calculate the median of the image pixel window.
- Step 7. If $abs(md - pc) < Th$, then replace the center pixel value with the calculated median value otherwise not.
- Step 8. Do the steps 5 – 7 until all the pixels are processed.
- Step 9. Calculate the intuitionistic fuzzy divergence measure, intuitionistic fuzzy entropy and cost value between original and all denoised images.
- Step 10. Choose the optimal threshold value as the minimum of the cost values.
- Step 11. Finally, the best denoised image corresponding to the optimal threshold is selected.

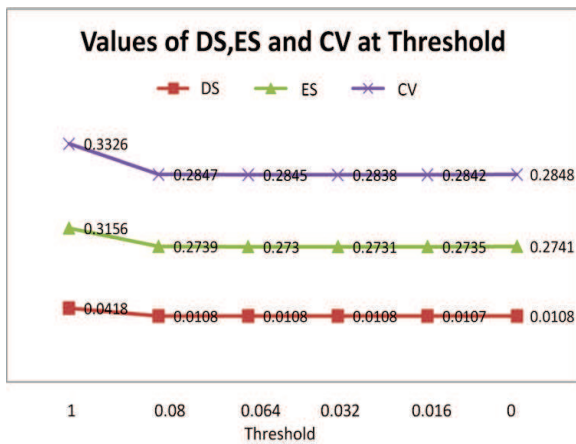


Fig. 2. Graph of DS, ES and CV

5. ANALYSIS OF RESULTS

Proposed algorithm change the noised pixels with the median of the window of size 3×3 , if the absolute difference of the median value and the center pixel is less than the predefined threshold. The problem is to choose a threshold (Th) as unexpected jumping of gray level in the algorithm to find the noised pixels which is related to the image. A new cost function is designed to find the optimal threshold in every image. The cost function is based on the intuitionistic fuzzy divergence measure of the denoised image \tilde{C} and original image \tilde{A} . The intuitionistic fuzzy entropy of denoised image \tilde{C} is added to the cost function. This is necessary, because when the algorithm threshold value is decreased, denoised image is blurred, although its divergence of original image is decreased. When the value of intuitionistic fuzzy entropy and intuitionistic fuzzy divergence measure are minimum, the sum that is the cost value will also decreased. It may be observed that the denoised image with a minimum cost value will be the best image. Experimental results with various values of hesitation constant c_1 , c_2 , β and selected threshold value Th are shown in figure 2. The salt and pepper noise of 20% is added into the original image for the result verification. After applying the various values of these parameters, we observed that the best denoised image is found, when $c_1 = 0.1$, $c_2 = 0.5$, $\beta = 0.99$.



Original Image

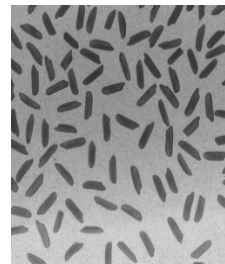


Fig. (a) Fuzzy Image

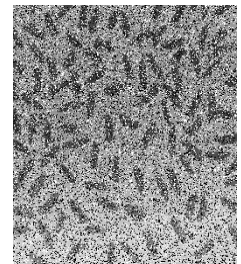


Fig. (b) Noised Image with 30% noise

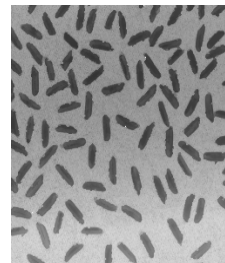


Fig. (c) Threshold = 0.08

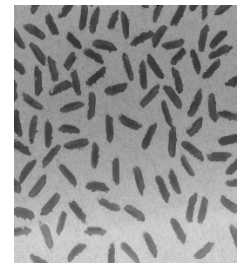


Fig. (d) Threshold = 0.064

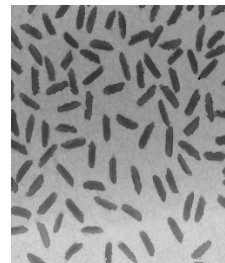


Fig. (e) Threshold = 0.048

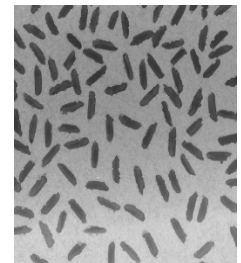


Fig. (f) Threshold = 0.32

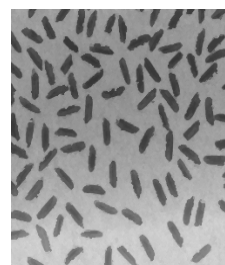


Fig. (g) Threshold = 0.16

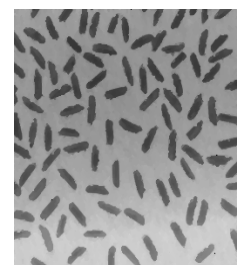


Fig. (h) Threshold = 0

Fig. 3. Rice Image



- A. The 'Rice' gray scale image taken of size 256×256 , is shown in figure 3. We denoised the image for the different values of threshold ranging from 0.08 to 0 with the parameters $c_1 = 0.1$, $c_2 = 0.5$, $\beta = 0.99$ and the corresponding values of cost function (CV) were obtained as shown in figure 2. It may be observed that the cost function value (CV) is minimum when the threshold value Th is 0.064. It can also be observed that the denoised image 3 (d) is better at $Th=0.064$, where the cost is minimum.
- B. It has been observed by the proposed method, if we add noise in the range of 55–80% than the results are better than median filter available in matlab. However, some fine details were removed because median filtering can't tell the difference between noise and fine details. Moreover, if the noise added to the original image is in between 10 – 45% the results are found to be better.

6. CONCLUSION

In this paper, we proposed an algorithm to find the optimal threshold value for denoising an image. In order to find the optimal threshold value to denoising an image, a new cost function is designed which is the combination of intuitionistic fuzzy divergence measure of the denoised image and original image and intuitionistic fuzzy entropy of denoised image. The proposed method has provided wonderful results as compared with other methods taking uncertainty into consideration. In future, the proposed method can be applied in the field of medical imaging and satellite image processing.

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