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Signed Dominating and Total Dominating Functions of Corona Product Graph of a Path with a Star

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ABSTRACT

Domination in graphs has been an extensively researched branch of graph theory. Graph theory is one of the most flourishing branches of modern mathematics and computer applications. An introduction and an extensive overview on domination in graphs and related topics is surveyed and detailed in the two books by Haynes et al. [1, 2]. Recently dominating functions in domination theory have received much attention. In this paper we present some results on minimal signed dominating functions and minimal total signed dominating functions of corona product graph of a path with a star.

Keywords

Corona Product, signed dominating function, Total signed dominating function.

Subject Classification: 68R10

1. INTRODUCTION

Domination Theory is an important branch of Graph Theory that has a wide range of applications to many fields like Engineering, Communication Networks, Social sciences, linguistics, physical sciences and many others. Allan, R.B. and Laskar, R.[3], Cockayne, E.J. and Hedetniemi, S.T. [4] have studied various domination parameters of graphs. Recently, dominating functions in domination theory have received much attention.

The concept of Signed dominating function was introduced by Dunbar et al. [5]. There is a variety of possible applications for this variation of domination. By assigning the values -1 or $+1$ to the vertices of a graph we can model such things as networks of positive and negative electrical charges, networks of positive and negative spins of electrons, and networks of people or organizations in which global decisions must be made. Zelinka, B.[6] introduced the concept of total signed dominating function of a graph.

Frucht and Harary [7] introduced a new product on two graphs G_1 and G_2, called corona product denoted by $G_1 \odot G_2$. The object is to construct a new and simple operation on two graphs G_1 and G_2 called their corona, with the property that the group of the new graph is in general isomorphic with the wreath product of the groups of G_1 and of G_2 .

The authors have studied some dominating functions of corona product graph of a cycle with a complete graph [8] and published papers on minimal dominating functions, some variations of Y – dominating functions and Y – total dominating functions [9,10,11,12,13].

In this paper we proved some results on signed dominating functions and total signed dominating functions of corona product graph of a path with a star.

2. CORONA PRODUCT OF P_n AND $K_{1,m}$

The **corona product** of a path P_n with star $K_{1,m}$ is a graph obtained by taking one copy of a n – vertex path P_n and n copies of $K_{1,m}$ and then joining the i^{th} vertex of P_n to every vertex of i^{th} copy of $K_{1,m}$ and it is denoted by $P_n \odot K_{1,m}$.

We require the following theorem whose proof can be found in Siva Parvathi, M. [8].

Theorem 2.1: The degree of a vertex v_i in $G = P_n \odot K_{1,m}$ is given by

$$d(v_i) = \begin{cases} m+3, & if \quad v_i \in P_n \ \ and \quad 2 \le i \le (n-1), \\ m+2, & if \quad v_i \in P_n \ \ and \qquad i = 1 \ \ or \ \ n, \\ m+1, & if \ \ v_i \in K_{1,m} \ and \ \ v_i \ \ is\ in\ first\ partition, \\ 2, & if \ \ v_i \in K_{1,m} \ and \ \ v_i \ \ is\ in\ second\ partition. \end{cases}$$

3. SIGNED DOMINATING FUNCTIONS

In this section we prove some results on minimal signed dominating functions of the graph $G = P_n \odot K_{1,m}$. Let us recall the definitions of signed dominating function and minimal signed dominating function of a graph $G(V, E)$.

Definition: Let $G\,(V, E\,)$ be a graph. A function $f : V \rightarrow \{-1\,, 1\}$ is called a **signed dominating function** (SDF) of G if $f(N[v]) = \sum_{u \in N[v]} f(u) \geq 1,\ for\ each\ \ v \in V.$

A signed dominating function f of G is called a **minimal signed dominating function** (MSDF) if for all $g < f, g$ is not a signed dominating function.

Theorem 3.1: A function $f : V \rightarrow \{-1, 1\}$ defined by

$$f(v) = \begin{cases} -1, & \text{for } \left\lfloor \frac{m+1}{2} \right\rfloor \text{ vertices in each copy of } K_{1,m} \text{ whose degree is 2 in G}, \\ 1, & \text{otherwise.} \end{cases}$$

is a minimal signed dominating function of $G = P_n \odot K_{1,m}$.

Proof: Let f be a function defined as in the hypothesis.

Case I: Suppose m is even.

Then $\left\lfloor \frac{m+1}{2} \right\rfloor = \frac{m}{2}$.

By the definition of the function, -1 is assigned to $\frac{m}{2}$ vertices in each copy of $K_{1,m}$ whose degree is 2 and 1 is assigned to $\frac{m}{2}$+1 vertices in each copy of $K_{1,m}$ in G. Also 1 is assigned to the vertices of P_n in G.

Case 1: Let $v \in P_n$ be such that $d(v) = m + 3$ in G.

Then $N[v]$ contains $m + 1$ vertices of $K_{1,m}$ and three vertices of P_n in G.

So $\sum_{u \in N[v]} f(u) = 1+1+1+\left[\frac{m}{2}(-1)+\left(\frac{m}{2}+1\right)(1)\right] = 3-\frac{m}{2}+\frac{m}{2}+1 = 4.$

Case 2: Let $v \in P_n$ be such that $d(v) = m + 2$ in G.

Then $N[v]$ contains $m + 1$ vertices of $K_{1,m}$ and two vertices of P_n in G.

So $\sum_{u \in N[v]} f(u) = 1+1+\left[\frac{m}{2}(-1)+\left(\frac{m}{2}+1\right)(1)\right] = 2-\frac{m}{2}+\frac{m}{2}+1 = 3.$

Case 3: Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G.

Then $N[v]$ contains $m + 1$ vertices of $K_{1,m}$ and one vertex of P_n in G.

So $\sum_{u \in N[v]} f(u) = 1+\left[\frac{m}{2}(-1)+\left(\frac{m}{2}+1\right)(1)\right] = 1-\frac{m}{2}+\frac{m}{2}+1 = 2.$

Case 4: Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G.

Then $f(v) = -1$ or $f(v) = 1$.

Now $N[v]$ contains two vertices of $K_{1,m}$ and one vertex of P_n in G.

If $f(v) = -1$, then $\sum_{u \in N[v]} f(u) = 1+1+(-1) = 1.$

If $f(v) = 1$, then $\sum_{u \in N[v]} f(u) = 1+1+1 = 3.$

Therefore for all possibilities,

we get $\sum_{u \in N[v]} f(u) \geq 1, \quad \forall \ v \in V.$

This implies that f is a SDF.

Now we check for the minimality of f.

Define $g : V \rightarrow \{ -1, 1 \}$ by

$$g(v) = \begin{cases} -1, & \text{for one vertex } v_k \text{ of } P_n \text{ in G,} \\ -1, & \text{for } \frac{m}{2} \text{ vertices in each copy of } K_{1,m} \text{ whose degree is 2 in G,} \\ 1, & \text{otherwise.} \end{cases}$$

Case (i): Let $v \in P_n$ be such that $d(v) = m + 3$ in G.

Sub case 1: Let $v_k \in N[v]$.

Then $\sum_{u \in N[v]} g(u) = (-1)+1+1+\left[\frac{m}{2}(-1)+\left(\frac{m}{2}+1\right)(1)\right] = 1-\frac{m}{2}+\frac{m}{2}+1 = 2.$

Sub case 2: Let $v_k \notin N[v]$.

Then $\sum_{u \in N[v]} g(u) = 1+1+1+\left[\frac{m}{2}(-1)+\left(\frac{m}{2}+1\right)(1)\right] = 3-\frac{m}{2}+\frac{m}{2}+1 = 4.$

Case (ii): Let $v \in P_n$ be such that $d(v) = m + 2$ in G.

Sub case 1: Let $v_k \in N[v]$.

Then $\sum_{u \in N[v]} g(u) = (-1)+1+\left[\frac{m}{2}(-1)+\left(\frac{m}{2}+1\right)(1)\right] = 0-\frac{m}{2}+\frac{m}{2}+1 = 1.$

Sub case 2: Let $v_k \notin N[v]$.

Then $\sum_{u \in N[v]} g(u) = 1+1+\left[\frac{m}{2}(-1)+\left(\frac{m}{2}+1\right)(1)\right] = 2-\frac{m}{2}+\frac{m}{2}+1 = 3.$

Case (iii): Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G.

Sub case 1: Let $v_k \in N[v]$.

Then $\sum_{u \in N[v]} g(u) = (-1)+\left[\frac{m}{2}(-1)+\left(\frac{m}{2}+1\right)(1)\right] = (-1)-\frac{m}{2}+\frac{m}{2}+1 = 0.$

Sub case 2: Let $v_k \notin N[v]$.

Then $\sum_{u \in N[v]} g(u) = 1+\left[\frac{m}{2}(-1)+\left(\frac{m}{2}+1\right)(1)\right] = 1-\frac{m}{2}+\frac{m}{2}+1 = 2.$

Case (iv): Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G.

Then $g(v) = -1$ or $g(v) = 1$.

Sub case 1: Let $v_k \in N[v]$.

If $g(v) = -1$, then $\sum_{u \in N[v]} g(u) = (-1)+1+(-1) = -1.$

If $g(v) = 1$, then $\sum_{u \in N[v]} g(u) = (-1)+1+1 = 1.$

Sub case 2: Let $v_k \notin N[v]$.

If $g(v) = -1$, then $\sum_{u \in N[v]} g(u) = 1+1+(-1) = 1.$

If $g(v) = 1$, then $\sum_{u \in N[v]} g(u) = 1+1+1 = 3.$

This implies that $\sum_{u \in N[v]} g(u) < 1$, for some $v \in V$.

So g is not a SDF.

Since g is defined arbitrarily, it follows that there exists no $g < f$ such that g is a SDF.

Therefore f is a MSDF.

Case II: Suppose m is odd.

Then $\left\lfloor \frac{m+1}{2} \right\rfloor = \frac{m+1}{2}$.

By the definition of the function, -1 is assigned to $\frac{m+1}{2}$ vertices in each copy of $K_{1,m}$ whose degree is 2 and 1 is assigned to $\frac{m+1}{2}$ vertices in each copy of $K_{1,m}$ in G respectively. Also 1 is assigned to the vertices of P_n in G.

Case 1: Let $v \in P_n$ be such that $d(v) = m + 3$ in G.

So $\sum_{u \in N[v]} f(u) = 1+1+1+\left[\left(\frac{m+1}{2}\right)(-1)+\left(\frac{m+1}{2}\right)(1)\right] = 3 - \frac{m}{2} - \frac{1}{2} + \frac{m}{2} + \frac{1}{2} = 3.$

Case 2: Let $v \in P_n$ be such that $d(v) = m + 2$ in G.

So

$$\sum_{u \in N[v]} f(u) = 1+1+\left[\left(\frac{m+1}{2}\right)(-1)+\left(\frac{m+1}{2}\right)(1)\right] = 2 - \frac{m}{2} - \frac{1}{2} + \frac{m}{2} + \frac{1}{2} = 2.$$

Case 3: Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G.

So

$$\sum_{u \in N[v]} f(u) = 1+\left[\left(\frac{m+1}{2}\right)(-1)+\left(\frac{m+1}{2}\right)(1)\right] = 1 - \frac{m}{2} - \frac{1}{2} + \frac{m}{2} + \frac{1}{2} = 1.$$

Case 4: Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G.

Then $f(v) = -1$ or $f(v) = 1$.

If $f(v) = -1$, then $\sum_{u \in N[v]} f(u) = 1+1+(-1) = 1.$

If $f(v) = 1$, then $\sum_{u \in N[v]} f(u) = 1+1+1 = 3.$

Therefore for all possibilities,

we get $\sum_{u \in N[v]} f(u) \geq 1, \forall \ v \in V.$

This implies that f is a SDF.

Now we check for the minimality of f.

Define $g : V \rightarrow \{ -1, 1 \}$ by

$$g(v) = \begin{cases} -1, & \text{for one vertex } v_k \text{ of } P_n \text{ in G,} \\ -1, & \text{for } \left(\frac{m+1}{2}\right) \text{ vertices in each copy of } K_{1,m} \text{ whose degree is 2 in G,} \\ 1, & \text{otherwise.} \end{cases}$$

Case (i): Let $v \in P_n$ be such that $d(v) = m + 3$ in G.

Sub case 1: Let $v_k \in N[v]$.

Then

$$\sum_{u \in N[v]} g(u) = (-1)+1+1+\left[\left(\frac{m+1}{2}\right)(-1)+\left(\frac{m+1}{2}\right)(1)\right] = 1 - \frac{m}{2} - \frac{1}{2} + \frac{m}{2} + \frac{1}{2} = 1.$$

Sub case 2: Let $v_k \notin N[v]$.

Then

$$\sum_{u \in N[v]} g(u) = 1+1+1+\left[\left(\frac{m+1}{2}\right)(-1)+\left(\frac{m+1}{2}\right)(1)\right] = 3 - \frac{m}{2} - \frac{1}{2} + \frac{m}{2} + \frac{1}{2} = 3.$$

Case (ii): Let $v \in P_n$ be such that $d(v) = m + 2$ in G.

Sub case 1: Let $v_k \in N[v]$.

Then

$$\sum_{u \in N[v]} g(u) = (-1)+1+\left[\left(\frac{m+1}{2}\right)(-1)+\left(\frac{m+1}{2}\right)(1)\right] = 0 - \frac{m}{2} - \frac{1}{2} + \frac{m}{2} + \frac{1}{2} = 0.$$

Sub case 2: Let $v_k \notin N[v]$.

Then

$$\sum_{u \in N[v]} g(u) = 1+1+\left[\left(\frac{m+1}{2}\right)(-1)+\left(\frac{m+1}{2}\right)(1)\right] = 2 - \frac{m}{2} - \frac{1}{2} + \frac{m}{2} + \frac{1}{2} = 2.$$

Case (iii): Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G.

Sub case 1: Let $v_k \in N[v]$.

Then

$$\sum_{u \in N[v]} g(u) = (-1)+\left[\left(\frac{m+1}{2}\right)(-1)+\left(\frac{m+1}{2}\right)(1)\right] = (-1) - \frac{m}{2} - \frac{1}{2} + \frac{m}{2} + \frac{1}{2} = -1.$$

Sub case 2: Let $v_k \notin N[v]$.

Then

$$\sum_{u \in N[v]} g(u) = 1+\left[\left(\frac{m+1}{2}\right)(-1)+\left(\frac{m+1}{2}\right)(1)\right] = 1 - \frac{m}{2} - \frac{1}{2} + \frac{m}{2} + \frac{1}{2} = 1.$$

Case (iv): Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G.

Then $g(v) = -1$ or $g(v) = 1$.

Sub case 1: Let $v_k \in N[v]$.

If $g(v) = -1$, then $\sum_{u \in N[v]} g(u) = (-1)+1+(-1) = -1.$

If $g(v) = 1$, then $\sum_{u \in N[v]} g(u) = (-1)+1+1 = 1.$

Sub case 2: Let $v_k \notin N[v]$.

If $g(v) = -1$, then $\sum_{u \in N[v]} g(u) = 1+1+(-1) = 1.$

If $g(v) = 1$, then $\sum_{u \in N[v]} g(u) = 1+1+1 = 3.$

This implies that $\sum_{u \in N[v]} g(u) < 1$, for some $v \in V$.

So g is not a SDF.

Since g is defined arbitrarily, it follows that there exists no $g < f$ such that g is a SDF.

Therefore f is a MSDF. ∎

4. TOTAL SIGNED DOMINATING FUNCTIONS

In this section we discuss total signed dominating functions and minimal total signed dominating functions of graph $G = P_n \odot K_{1,m}$. First we recall the definitions of total signed dominating function of a graph.

Definition: Let $G\ (V, E\ )$ be a graph. A function $f : V \rightarrow \{ -1,\ 1\}$ is called a **total signed dominating function**

(TSDF) of G if

$$f(N(v)) = \sum_{u\in N(v)} f(u) \geq 1, \quad for\ each\ v \in V.$$

A total signed dominating function f of G is called a **minimal total signed dominating function** (MTSDF) if for all $g < f$, g is not a total signed dominating function.

Theorem 4.1: A function f : V → { -1, 1} defined by

$$f(v) = \begin{cases} -1, & \text{for } \left\lfloor \frac{m}{2} \right\rfloor \text{ vertices of } K_{1,m} \text{ whose degree is 2 in each copy in G,} \\ 1, & \text{otherwise.} \end{cases}$$

is a Minimal Total Signed Dominating Function of $G = P_n \odot K_{1,m}$.

Proof: Consider the graph $G = P_n \odot K_{1,m}$ with vertex set V.

Let f be a function defined as in the hypothesis.

Case I: Suppose m is even.

Then $\left\lfloor \frac{m}{2} \right\rfloor = \frac{m}{2}$.

By the definition of the function, -1 is assigned to $\frac{m}{2}$ vertices of $K_{1,m}$ whose degree is 2 in each copy in G and 1 is assigned to $\frac{m}{2}$+1 vertices of $K_{1,m}$ in each copy in G. Also 1 is assigned to the vertices of P_n in G.

Case 1: Let $v \in P_n$ be such that $d(v) = m + 3$ in G.

Then $N(v)$ contains $m + 1$ vertices of $K_{1,m}$ and two vertices of P_n in G.

So $$\sum_{u\in N(v)} f(u) = 1+1+\left[\frac{m}{2}(-1)+\left(\frac{m}{2}+1\right)(1)\right] = 2-\frac{m}{2}+\frac{m}{2}+1 = 3.$$

Case 2: Let $v \in P_n$ be such that $d(v) = m + 2$ in G.

Then $N(v)$ contains $m + 1$ vertices of $K_{1,m}$ and one vertex of P_n in G.

So $$\sum_{u\in N(v)} f(u) = 1+\left[\frac{m}{2}(-1)+\left(\frac{m}{2}+1\right)(1)\right] = 1-\frac{m}{2}+\frac{m}{2}+1 = 2.$$

Case 3: Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G.

Then $N(v)$ contains m vertices of $K_{1,m}$ whose degree is 2 and one vertex of P_n in G.

So $$\sum_{u\in N(v)} f(u) = 1+\left[\frac{m}{2}(-1)+\left(\frac{m}{2}\right)(1)\right] = 1-\frac{m}{2}+\frac{m}{2} = 1.$$

case 4: Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G.

Then $N(v)$ contains one vertex of $K_{1,m}$ whose degree is $m + 1$ and one vertex of P_n in G.

So $$\sum_{u\in N(v)} f(u) = 1+1 = 2.$$

Therefore for all possibilities,

we get $$\sum_{u\in N(v)} f(u) \geq 1, \quad \forall \ v \in V.$$

This implies that f is a Total Signed Dominating Function.

Now we check for the minimality of f.

Define g : V → { -1, 1} by

$$g(v) = \begin{cases} -1, & \text{for one vertex } v_k \text{ of } P_n \text{ in G,} \\ -1, & \text{for } \frac{m}{2} \text{ vertices of } K_{1,m} \text{ whose degree is 2 in each copy in G,} \\ 1, & \text{otherwise.} \end{cases}$$

Case (i): Let $v \in P_n$ be such that $d(v) = m + 3$ in G.

Sub case 1: Let $v_k \in N(v)$.

Then $$\sum_{u\in N(v)} g(u) = (-1)+1+\left[\frac{m}{2}(-1)+\left(\frac{m}{2}+1\right)(1)\right] = 0-\frac{m}{2}+\frac{m}{2}+1 = 1.$$

Sub case 2: Let $v_k \notin N(v)$.

Then $$\sum_{u\in N(v)} g(u) = 1+1+\left[\frac{m}{2}(-1)+\left(\frac{m}{2}+1\right)(1)\right] = 2-\frac{m}{2}+\frac{m}{2}+1 = 3.$$

Case (ii): Let $v \in P_n$ be such that $d(v) = m + 2$ in G.

Sub case 1: Let $v_k \in N(v)$.

Then $$\sum_{u\in N(v)} g(u) = (-1)+\left[\frac{m}{2}(-1)+\left(\frac{m}{2}+1\right)(1)\right] = (-1)-\frac{m}{2}+\frac{m}{2}+1 = 0.$$

Sub case 2: Let $v_k \notin N(v)$.

Then $$\sum_{u\in N(v)} g(u) = 1+\left[\frac{m}{2}(-1)+\left(\frac{m}{2}+1\right)(1)\right] = 1-\frac{m}{2}+\frac{m}{2}+1 = 2.$$

Case (iii): Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G.

Sub case 1: Let $v_k \in N(v)$.

Then $$\sum_{u\in N(v)} g(u) = (-1)+\left[\frac{m}{2}(-1)+\left(\frac{m}{2}\right)(1)\right] = (-1)-\frac{m}{2}+\frac{m}{2} = -1.$$

Sub case 2: Let $v_k \notin N(v)$.

Then $$\sum_{u\in N(v)} g(u) = 1+\left[\frac{m}{2}(-1)+\left(\frac{m}{2}\right)(1)\right] = 1-\frac{m}{2}+\frac{m}{2} = 1.$$

Case (iv): Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G.

Sub case 1: Let $v_k \in N(v)$.

Then $$\sum_{u\in N(v)} g(u) = (-1)+1 = 0.$$

Sub case 2: Let $v_k \notin N[v]$.

Then $\sum_{u \in N(v)} g(u) = 1 + 1 = 2.$

From the above cases, it implies that $\sum_{u \in N(v)} g(u) < 1,$

for some v $\in$ V.

So g is not a TSDF.

Since g is defined arbitrarily, it follows that there exists no $g < f$ such that g is a TSDF.

Thus f is a MTSDF.

Case II: Suppose m is odd. Then $\left\lfloor \frac{m}{2} \right\rfloor = \frac{m-1}{2}$.

By the definition of the function, -1 is assigned to $\frac{m-1}{2}$ vertices of $K_{1,m}$ whose degree is 2 in each copy in G and 1 is assigned to $\left(\frac{m+1}{2}\right) + 1$ vertices of $K_{1,m}$ in each copy in G.
Also 1 is assigned to the vertices of P_n *in G*.
Case 1: Let $v \in P_n$ be such that $d(v) = m + 3$ in G.

So
$$\sum_{u \in N(v)} f(u) = 1 + 1 + \left[\left(\frac{m-1}{2}\right)(-1) + \left(\frac{m+1}{2} + 1\right)(1)\right] = 2 - \frac{m}{2} + \frac{1}{2} + \frac{m}{2} + \frac{1}{2} + 1 = 4.$$

Case 2: Let $v \in P_n$ be such that $d(v) = m + 2$ in G.

So
$$\sum_{u \in N(v)} f(u) = 1 + \left[\left(\frac{m-1}{2}\right)(-1) + \left(\frac{m+1}{2} + 1\right)(1)\right] = 1 - \frac{m}{2} + \frac{1}{2} + \frac{m}{2} + \frac{1}{2} + 1 = 3.$$
Case 3: Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G.

So
$$\sum_{u \in N(v)} f(u) = 1 + \left[\left(\frac{m-1}{2}\right)(-1) + \left(\frac{m+1}{2} + 1\right)(1)\right] = 1 - \frac{m}{2} + \frac{1}{2} + \frac{m}{2} + \frac{1}{2} + 1 = 3.$$
Case 4: Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G.

So $\sum_{u \in N(v)} f(u) = 1 + 1 = 2.$

Therefore for all possibilities,

we get $\sum_{u \in N(v)} f(u) \geq 1, \forall$ v $\in$ V.

This implies that f is a Total Signed Dominating Function.

Now we check for the minimality of f.

Define g : V → { -1, 1} by

$$g(v) = \begin{cases} -1, & \text{for one vertex } v_k \text{ of } P_n \text{ in } G, \\ -1, & \text{for} \left(\frac{m-1}{2}\right) \text{ vertices of } K_{1,m} \text{ whose degree is 2 in each copy in } G, \\ 1, & \text{otherwise.} \end{cases}$$

Case (i): Let $v \in P_n$ be such that $d(v) = m + 3$ in G.

Sub case 1: Let $v_k \in N(v)$.

Then
$$\sum_{u \in N(v)} g(u) = (-1) + 1 + \left[\left(\frac{m-1}{2}\right)(-1) + \left(\frac{m+1}{2} + 1\right)(1)\right] = 0 - \frac{m}{2} + \frac{1}{2} + \frac{m}{2} + \frac{1}{2} + 1 = 2.$$
Sub case 2: Let $v_k \notin N(v)$.

Then
$$\sum_{u \in N(v)} g(u) = 1 + 1 + \left[\left(\frac{m-1}{2}\right)(-1) + \left(\frac{m+1}{2} + 1\right)(1)\right] = 2 - \frac{m}{2} + \frac{1}{2} + \frac{m}{2} + \frac{1}{2} + 1 = 4.$$
Case (ii): Let $v \in P_n$ be such that $d(v) = m + 2$ in G.

Sub case 1: Let $v_k \in N(v)$.

Then
$$\sum_{u \in N(v)} g(u) = (-1) + \left[\left(\frac{m-1}{2}\right)(-1) + \left(\frac{m+1}{2} + 1\right)(1)\right] = (-1) - \frac{m}{2} + \frac{1}{2} + \frac{m}{2} + \frac{1}{2} + 1 = 1.$$
Sub case 2: Let $v_k \notin N(v)$.

Then
$$\sum_{u \in N(v)} g(u) = 1 + \left[\left(\frac{m-1}{2}\right)(-1) + \left(\frac{m+1}{2} + 1\right)(1)\right] = 1 - \frac{m}{2} + \frac{1}{2} + \frac{m}{2} + \frac{1}{2} + 1 = 3.$$
Case (iii): Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G.

Sub case 1: Let $v_k \in N(v)$.

Then
$$\sum_{u \in N(v)} g(u) = (-1) + \left[\left(\frac{m-1}{2}\right)(-1) + \left(\frac{m+1}{2}\right)(1)\right] = (-1) - \frac{m}{2} + \frac{1}{2} + \frac{m}{2} + \frac{1}{2} = 0.$$
Sub case 2: Let $v_k \notin N(v)$.

Then
$$\sum_{u \in N(v)} g(u) = 1 + \left[\left(\frac{m-1}{2}\right)(-1) + \left(\frac{m+1}{2}\right)(1)\right] = 1 - \frac{m}{2} + \frac{1}{2} + \frac{m}{2} + \frac{1}{2} = 2.$$

Case (iv): Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G.

Sub case 1: Let $v_k \in N[v]$.

Then $\sum_{u \in N(v)} g(u) = (-1) + 1 = 0.$

Sub case 2: Let $v_k \notin N(v)$.

Then $\sum_{u \in N(v)} g(u) = 1 + 1 = 2.$

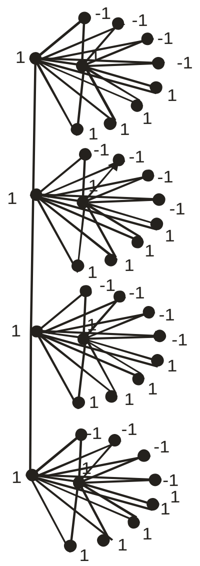
This implies that $\sum_{u \in N(v)} g(u) < 1$, for some v$\in$V.

So g is not a TSDF.
Since g is defined arbitrarily, it follows that there exists no $g < f$ such that g is a TSDF.
Thus f is a MTSDF.

5. CONCLUSION

It is interesting to study the signed and total signed dominating functions of the corona product graph of a path with a star. This work gives the scope for the study of convexity of these functions and the authors have also studied this concept.

6. ILLUSTRATION



$G = P_4 \odot K_{1,8}$

The function f takes the value -1 for $\frac{m}{2}$ vertices in each copy of $K_{1,8}$ whose degree is 2 and the value 1 for the remaining vertices in G.

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