Complementary Perfect Domination Number of Regular Graphs

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ABSTRACT

In this paper authors describe domination number of regular graphs as well as complimentary perfect domination number and induced complimentary perfect domination number, denoted by cpd and iced. Let G(v,e) be a graph with 'n' vertices and 'e' edges then these are denoted by $X_{cp}(G)$ .and $X_{icp}(G)$ .In this paper we describe, How to calculate $X_{cp}(G)$ .and $X_{icp}(G)$ of regular graphs. Authors characterize 2 regular graphs with X_{icp} and 3 regular graphs with X_{icp} and describe an upper limit for number of vertices in d-regular graph. In the end of the paper we characterize all the d-regular graphs with $X_{icp}(G)$ and practical utilization of cpd and ipcd.

Keywords

Domination number, representation of two graphs with one, cpd and ipcd of regular graph, d-regular.

1. INTRODUCTION

In this modern age of science and pacific root of computer life, so many scholars are doing hard work in the domination theory. In this paper we plan a structure of traffic control system with regular graphs and explain how it is used. Because of it more than lacks Rs. are spent on the flyover of a crossing.

Let G(v,e) be a regular graph with n vertices and e edges that means Graph is finite, simple undirected. The number of vertices is denoted by $V_1, V_2, V_3,. . . . . ,$Vn. A lot of work has been done on domination sets and domination number. Different type of domination sets and domination number has been introduced and studied by several authors. [1][4][5][7][10] Paulraj Joseph and Arumugam S have found the relationship between domination number and connectivity in graph as well as domination number and coloring in graphs.

In the same way Paulraj J, Joseph and Mehadevan [11] have introduced the concept of complimentary perfect domination number but they fail to calculate cpd and icpd of regular graphs.

Avadayappan and Senthil Kumar have proved that for any graph G and a vertex V of G.

$$X_{cp}(G_m(v)) = X_{cp}(G) + m$$, where $G_m(v)$ is the graph obtained from G by identification of the center vertex of a star graph $K_{i,m}$ at a vertex V.

1.1 Domination Sets

A subset S of vertices is known domination set of G if every vertex in V-S is adjacent to at least one vertex in S.

1.2 Domination Number

Domination number of a graph G is the minimum cardinality taken over all dominating set in G and denoted by X(G).

1.3 Complementary Perfect Domination Number

The complimentary perfect domination number of graph G is the minimum cardinality taken over all complementary perfect domination sets in g and denoted by $X_{cp}(G)$.

1.4 Induced Perfect Domination Number

The minimum cardinality taken over all the induced perfect Dominaiton sets of G is known as induced complementary perfect domination number of G and denoted by $X_{icp}(G)$. An induced complementary perfect domination set of a graph G (icpd-set) is a domination set of G such that induced sub graph(v-s) has only independent edges. Any ipcd-set with X_{icp} elements is known as X_{icp} -set of G.
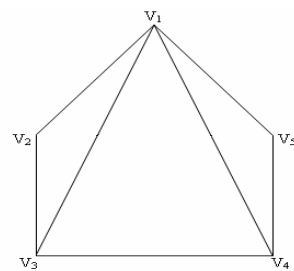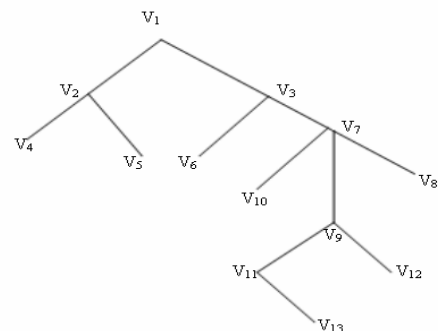

Fig1.1

Fig 1.2

In fig 1.1 $X_{cp}(G) = 1$ and $X_{icp} = 3$

Here S = { V_1 } is the X_{cp}-set and S = { V_1, V_2, V_3 } is a X_{icp}-set as well as fig 1.2 has $X_{cp} = 8$ and $X_{icp} = 9$.

Here S = { V_2, V_5, V_6, V_8, V_{10}, V_{12}, V_{13} } is X_{cp} - set and S = { V_3, V_5, V_6, V_8, V_{10}, V_{11}, V_{12}, V_{13} } is a X_{icp}-set.

2. NOTATIONS

G(v,e) → A graph with n vertices and e edges.
n-regular graph → A graph G is said to be n –regular if degree of Each vertex is n.
d(v) → Degree of vertex V in G.
Δ → Maximum degree in G.
$X_{cp}(G)$ → Complimentary perfect domination number of G.
$X_{icp}(G)$ → Induced complimentary perfect domination number of G.
[x] → Greatest number, which is at least x.
K_n → Complete graph with n vertices.
C_n → Cyclic on n vertices.
<S> or (S) → Induced sub graph of G.
R → Chromatic number of a graph.
$K_{3,3}$ → Bipartite graph.

2.1 Assumptions / Observation

1. For any Graph, $n \geq 2$ $X_{icp}(G) = n$ iff G is a star.

2. $X_{icp}(G) = 1$ $\Leftrightarrow$ G is isomorphic to $K_1 \vee mK_2$, $m \geq 1$

3. Any Icpd – set of G must contain all the Pendant vertex of G.

4. If G is not star then $\left[\dfrac{n}{\Delta+1}\right] \leq X_{icp} \leq n-2$

5. $X_{icp}(K_n) = n-2$ $\forall$ $n \geq 3$

3. COMPLEMENTARY PERFECT DOMINATION NUMBER OF REGULAR GRAPHS

This section authors try to fill some gap by solving some theorem and its characterization, all cycles of 2-regular graph with $X_{cp} = \alpha$ and 3-regular graphs with $X_{icp} = \alpha = 2$

Theorem 1.1

G is 2-regular graph with
$$X_{cp}(G) = r \Leftrightarrow G \cong c_n, n = 4,5,6,7,9$$

Proof: - Given that G is 2-regular i.e. 2-regular is a cycle.

⇒ Cycle may be even or odd

Even cycle have $r = 2$ and odd cycle have $r = 3$.

⇒ $X_{cp} = r$

⇒ X_{cp} = 2 or 3

Now let $X_{cp} = r = 2$ then

$$2 = X_{cp} \geq \left[\dfrac{n}{\Delta+1}\right] = \left[\dfrac{n}{3}\right]$$

Therefore $n \leq 6$

Hence $G \cong C_4$ or C_6

In the same way

$$X_{cp} = r = 3$$ Then

$$3 = V_{cp} \geq \left[\dfrac{n}{\Delta+1}\right] = \left[\dfrac{n}{3}\right]$$

⇒ $P \leq 9$ And $G \cong C_5$ or C_7 or C_9

Conversely,

$$G \cong C_n, n = 4,5,6,7,9$$

⇒ There exist some circuits of even and odd length.

⇒ G is 2-regular with $X_{cp} = r$

Theorem 1.2

G is a 3 regular graph with $X_{cp} = r = 2$ if and only if $G \cong K_{3,3}$

Proof: - First assuming that G is 3-regular graph with $X_{cp} = r = 2$ and prove that $G \cong K_{3,3}$

G be 3-regular graph with $X_{cp} = r = 2$ than

$$2 = X_{cp} \geq \left[\dfrac{n}{\Delta+1}\right] = \left[\dfrac{n}{3}\right]$$

⇒ $n \leq 8$
⇒ but G is 3-regular
⇒ n = 4, 5, 6 or 8
⇒ $n \neq 4$

Because K_4 is the only 3-regular graph on 4-vertices of which r = 4.

⇒ n = 6 or 8 i.e. there are 2 case.

Case I: - n = 6

Let V_1, V_2, V_3, V_4, V_5, V_6 be the six vertices of G.

Let S={ V_1, V_2 } be X_{cp} - set then <V-S> should have two independent edges say $V_3 V_4$ and $V_5 V_6$. Since r = 2, G has no triangle and hence both V_3 and V_4 or V_5 and V_6

cannot be adjacent to V_1 i.e. V_1 is adjacent to at most 2 non-adjacent vertices of V.S. but G is 3-regular with r =2 therefore V_1 is adjacent to V_2 and to exactly non-adjacent vertices of V-S say V_4 and V_6.

Similarly V_2 is Adjacent to V_1 and to exactly two non-adjacent vertices of V-S say V_3, V_5. Also a vertex in <V-S> cannot be adjacent to both V_1 and V_2. Keeping all above calculation the resulting graph is nothing but $K_{3,3}$ as shown in fig.
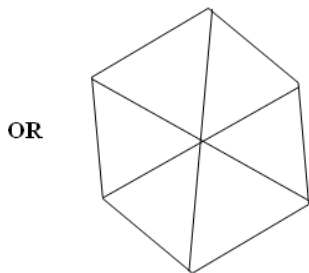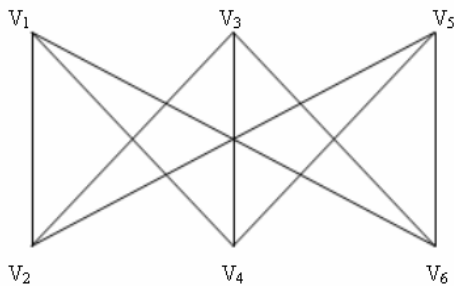
Case II: - If n = 8

Let $V_n \in G \ \forall \ n = 1,2,.....,8$ with $X_{cp} = 2$

Let S = { V_1, V_2 } be a $X_{cp} - set$. As G is A 3-reglarar graph and S is dominating set in V-S, V_1 and V_2 are adjacent to exactly three different vertices in V-S.
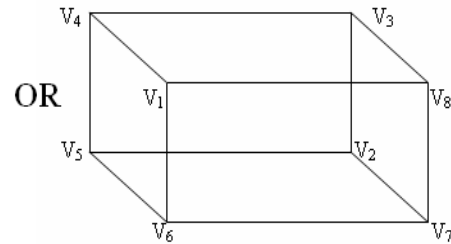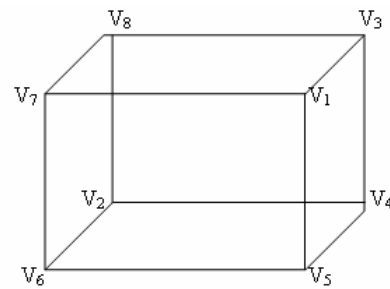
Let $$R(V_1) = \{V_3, V_5, V_7\}$$ and $R(V_2) = \{V_4, V_6, V_7\}$. Since r = 2, G has no triangle and V_3, V_5, V_7 are independent. Similarly V_4, V_6, V_8 are independent.

Now <V-S> is 2-regular bipartite graph with (U,V) Where U={V_3, V_5, V_7} and V={V_4, V_6, V_7, V_8} $\Rightarrow$ <V-S> is 2-regular with six vertices.

$$\Rightarrow \ <V - S> \cong C_6$$

Here resulting graph shown as

conversely, by theorem we know that r=3 of $K_{3,3}$

r=m for $K_{3,3}$, m > b

4. INDUCED PERFECT DOMINATION NUMBER OF REGULAR GRAPH

In this section, Author characterizes all the d-regular graphs with minimum possible induced complimentary perfect domination number.

Theorem 1.3

If G is a d-regular graph then $X_{icp}(G) \geq d - 1$ and

$$n \leq X_{icp} \left(\frac{2d - 1}{d - 1} \right)$$

Proof: - Let S be a set $X_{icp} - set$ in G. then all the vertices in <V-S> have degree 1.

(From definition of X_{icp})

Since the Graph is d-regular every vertex in <V-S> is adjacent to d-1 vertices in S.

Therefore $X_{icp} \geq d - 1$. In 1998 [8] Slater P.J. proof that
$$dX_{icp} \geq (n - X_{icp})(d - 1) = nd - n - dX_{icp} + X_{icp}$$

i.e. $$X_{icp} \geq n \left(\frac{d - 1}{2d - 1} \right)$$

or $$n \leq X_{icp} \left(\frac{2d - 1}{d - 1} \right)$$

Let Gi denote any i-regular on d-1 vertices where $d \geq 2$ and $2 \leq i \leq d - 2$, both d and i have same property.

When i=0,1,2 and d-1, then

$$G_i \cong K_{d-1}^c, \left(\frac{d-1}{2}\right)K_2, C_{d-1}$$

and K_{d-1} respectively.

Theorem 1.4

A d-regular graph of a graph G with $X_{icp} = d-1$ iff

$$G \cong G_i V\left(\frac{d-1}{2}\right)k_2$$

where $0 \leq i \leq d-2$

Proof: - Let G be A d-regular graph with $X_{icp} = d-1$

$$G \cong G_i V\left(\frac{d-1}{2}\right)k_2$$

and prove that given that G is a d-regular with $X_{icp} = d-1$

$\Rightarrow$ <S> has d-1 vertices and <V-S> has only independent edges.

By Theorem 1.3, $n \leq 2d-1$ and sumce G is d-regular $n \geq d+1$

$\Rightarrow$ $d+1 \leq n \leq 2d-1$

If <V-S> has $\left(\frac{d-i}{2}\right)$ edges. Where $0 \leq i \leq d-2$ and both i & d have same property.

$\Leftrightarrow$ <V-S> is 1-regular graph with d-I vertices.

$\Leftrightarrow$ G is d-regular (Every vertex in <V-S) is adjacent to all the d-1 vertices in <S>)

$\Leftrightarrow$ Every vertex in S is adjacent to d-i vertices in V-S.

$\Leftrightarrow$ Every vertex in S is adjacent to exactly I vertices in S.

$\Leftrightarrow$ <S> is i-regular and S has d-1 vertices.

$\Leftrightarrow$ $<S> \cong G_i$

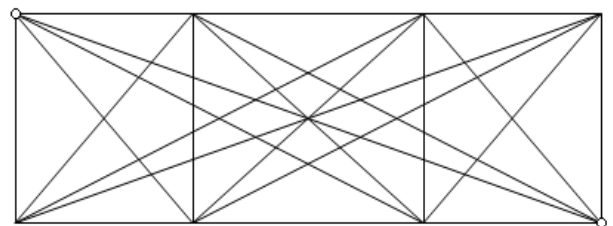$\Leftrightarrow$ $<S> \cong G_i \vee \left(\frac{d-i}{2}\right)K_2$

$\Leftrightarrow$

Illustration

1) From these theorem and calculations authors conclude that

2-regular graph with $X_{icp} = 1$

3-regular graph with $X_{icp} = 2$

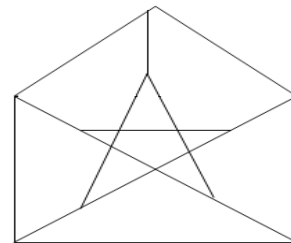4-regular graph with $X_{icp} = 3$

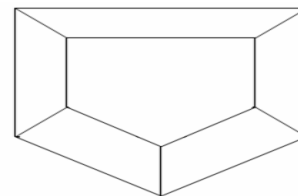5-regular graph with $X_{icp} = 4$
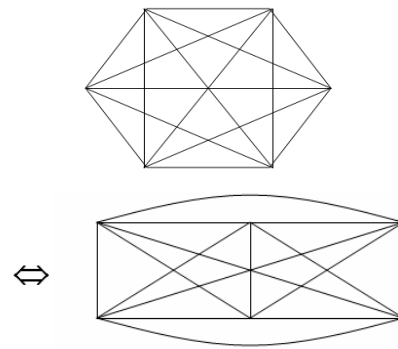
5-regular graph or K6

5-regular or 2K$_2$ ∨ 2K$_2$

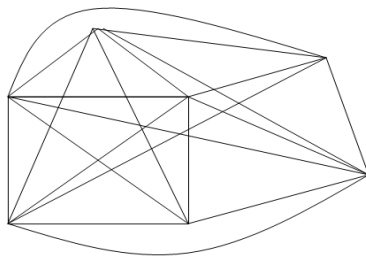2) Theorem 1.2 and Theorem 1.4 gives the help of detection of planarity of Peterson graph.
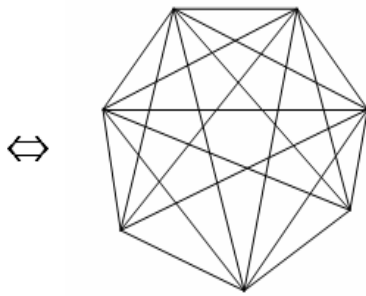

3-regular graph with n=10 is non-planner

It is also a 3-regular graph but not Peterson graph

3) 6-regular Graph with $\chi_{icp} = 5$

K₅ ∨ K₂

$$\Leftrightarrow$$

K₇

5. CONCLUSION

In the conclusion part of this paper, we can say that we have successfully describe domination number of regular graphs as well as complimentary perfect domination number and induced complimentary perfect domination number, denoted by cpd and iced. In future we will try to do more in graph theory to prove few more facts about graphs.

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